The Role of Contexts and Teacher’s Questioning to Enhance Students’ Thinking

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This paper discusses results from a design research in line with Realistic Mathematics Education (RME). Daily cycles of design, classroom experiments, and retrospective analysis are enacted in five days of working about division by fractions. Data consists of episodes of video classroom discussions, and samples of students’ work. The focus of discussion and analysis centres on the role of contexts and the role of teachers’ probing questions to elicit students’ thinking. Our findings suggest that contexts that are meaningful for and understandable by students bring out rich mathematical thinking and discussion amongst students. Meaningful contexts combined with teacher’s probing questions - highlighting big mathematical ideas - allow students to attain various approaches at different levels of formal mathematics.

Introduction

The role of contexts in learning mathematics has gained increased attention in the past few decades following a call for reform in teaching and learning mathematics. The reform movements in mathematics which underscore a move away from teaching mathematics as a series of abstract procedures, provides an impetus for contexts to enhance understanding of mathematical problems. Lave (1988) contends that every specific context has a potential to

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determine the choice of mathematical procedures. However as pointed out by Gravemeijer & Doorman (1999), contexts have been used more commonly as applications where students apply the mathematical ideas or properties to find solutions. Realistic Mathematics Education (RME) theory, inspired by Freudenthal’s (1983, 1991) idea of mathematics as a human activity, places contexts at a more crucial role. Contexts serve as a starting point whereby students explore and reinvent mathematical notions in a situation that is ‘experientially real’ for them (see e.g., Gravemeijer & Doorman, 1999). This is in line with Clarke’s (1997) outline of reform mathematics classroom characteristics whereby non-routine problems serve as a starting point and the focus of instruction without the provision of procedures for solving the problems.

Meaningful contexts afford a tool to assist students make sense and construct various strategies in working with the problem. Different characteristics of reformed mathematics classes require changes in teacher’s practice and beliefs, which are manifested in the teacher’s different roles in classroom practice. A number of studies have outlined and documented the changes in the teacher’s role (see e.g., Brown, Stein, & Forman, 1996; Clarke, 1997; Martin, McCrone, Bower, & Dindyal, 2005; Simon, 1994). Wood, Cobb and Yackel (1990) noted that the teacher’s role has shifted towards that of a facilitator of students’ learning rather than the authority and sole source of knowledge. In facilitating students’ learning process, teachers create a learning atmosphere whereby students’ ideas and solution serve as a basis for classroom discourse.

Current thinking in Indonesia, inspired by the Freudenthal’s RME, emphasises developing meaning and moving away from teaching based on rules. The movement to adapt RME to the Indonesian context called “PMRI” (Pendidikan Matematika Realistik Indonesia) began in 2000. A comprehensive summary of various elements of the movement has recently been published to mark a decade of PMRI movement (Sembiring, Hoogland, & Dolk, 2010). This approach entails a new teaching approach such as group work which encourages students to construct mathematical ideas together following RME’s teaching and learning tenets. In line with this approach, the teacher plays a role in facilitating students’ learning by utilising rich contextual problems, probing questions to guide students’ development of thinking and to lead classroom discussion. In examining how PMRI teaching and learning takes in place in the classroom, design research has been employed.
as a research tool. Collaborative effort between teacher educators and teachers to develop educational knowledge about how to create situations in which students can construct mathematical knowledge is one of key characteristics in design research. This paper will look into one of the outcomes of design research study, focusing on the mediating role of the teacher to support students’ learning. In contrast with the common practice in Indonesian classrooms where teachers tell students right or wrong answers, the design of activities aim to support students in developing their own thinking and share their thinking as the basis of classroom discourse.

**Situating the Research**

McClain and Cobb (2001) advocate a pro-active role for mathematics teachers to support students’ learning by developing positive classroom norms. A passive terminology about what a teacher should not do in the classroom is avoided. Instead, efforts to cultivate a positive, empowering view on the supporting role of the teacher are promoted. In line with this view, we try to encourage teacher educators and teachers to develop a pro-active role in the classroom in this study. For instance, instead of stating that a teacher should not give a judgment whether an answer is right or wrong, we support teachers in developing a habit of asking probing question in their reasoning, allowing students to decide for themselves whether an answer is right or wrong and whether their argumentation is accepted in class.

This role of the teacher can be described in terms of a didactical contract, and in terms of social and socio-mathematical norms in the classroom. Brousseau (1997) contends that teachers and students need to have a didactical contract (contract didactique) that requires a productive interaction that demands reciprocal obligations. The notion of the didactical contract is defined by Brousseau as follows:

... in all didactical situations, the teacher attempts to tell the students what she wants them to do. Theoretically the transition from the information and the teacher’s instruction to the expected answer must require students to bring the target knowledge into play. We know, the only way to “do” mathematics is to investigate and solve certain specific problems and, on this occasion, to raise new questions. The teacher must therefore arrange not the communication of knowledge, but the devolution of a good problem. If this devolution takes place, the students enter into the game and if they win, learning occurs. (Brousseau, 1997, p. 32)
At the level of the mathematics classroom, the didactical contract describes the didactic relations between teachers, their students and the mathematical investigations for a sequence of lessons. Such a didactical contract exists under all circumstances. Traditionally, in Indonesia the teacher transmits knowledge in the form of rules and tricks and learners have to practice those rules with straightforward and simple problems. The teacher classifies each answer as right or wrong and gives students some more practice. If there is any mathematical thinking happening in the classroom, it is often the teacher who is doing that thinking. Students are not really invited to think. Changing this traditional, unwritten contract is not easy as it requires changes in the roles of the teacher and learner, changes in the norms for participation, as well as redefining what constitutes mathematics. In all cases the teacher and the students have to negotiate what will happen in the classroom. In other words, the norms for participation in the mathematical discourse (Cobb, 1999; Cobb & Yackel, 1996) and the roles of the research team need to be changed.

One norm of participation promoted in the mathematics classroom is that students are not only expected to give answers, but also to publicly explain, justify, and defend their reasoning. This immediately requires students to listen to each other and to understand and to examine other students’ reasoning. One of the changes in the teacher’s role is the shift from transmitting knowledge and requiring practice through simple, one-step problems with right or wrong answers to inviting students to investigate mathematics and discuss their findings and understanding. It means a transition from judging answers as right or wrong to developing learners’ abilities to articulate emergent understandings as they engage in investigative activities. Changing these norms also requires a change in definition of mathematics and mathematical practice.

Methodology

Design research (Cobb, Stephan, McClain, & Gravemeijer, 2001; Edelson, 2002; Gravemeijer, 2004; Kelly, 2003; Research Advisory Committee, 1996) is an emerging paradigm which aims to develop sequences of activities and to grasp an empirically grounded understanding of how learning works. The iterative cyclic character of design research and its role in developing domain specific theories are the shared key characteristics of the design research. We contend that our interpretation of design research is a
combination of design research (Gravemeijer & Cobb, 2006), Simon’s (1995) notion of Hypothetical Learning Trajectory (HLT) and Lesson Study (Watanabe, 2002; Yoshida, 1999) as discussed in Widjaja and Dolk (2010). The analysis of classroom episodes and activities were conducted with interpretive lenses, focusing on how contexts and teacher’s questioning mediate and enhance students’ thinking.

In this paper, we will report a study of enacting daily cycles of design, classroom experiments, and retrospective analysis of activities in a five-day workshop on division by fractions. Data reported in this paper is a subset of data from design research cycles on division of fractions. Table 1 presents a summary of activities involved in the five-day workshop. We will report and analyse episodes of video classroom discussions, and samples of students’ work. Our analysis will be centred on the role of contexts and teacher’s questioning to enhance students’ thinking. The investigation that served as a starting point for Grade 5 class is given in Figure 1 whereas the description and discussion of these daily mini-cycles will be explicated in the next section.

A family buys 25 kilograms of rice and eats $\frac{3}{4}$ of a kilo each day. How many days can 25 kilograms of rice lasts for?

Figure 1. Rice problem as a starting point for discussion.

During the five days, the research team consisting of teacher educators and teachers designed the investigation, observed the class at work, discussed their observations, analysed the data, and planned the following day’s investigation. The resident teacher was present and worked closely together with the research team during all these steps. On a daily basis the research team kept to the following mini design cycle:

1. Design: planning an investigation, anticipatory thought experiment depicting what might happen in class, and predicting students’ thinking and what the teacher might do to help the growth and development of the children. When possible, the research team connected this planning and anticipating to their existing knowledge by formulating a conjectured theory.
2. Classroom teaching in which the children investigate the problem supported by the teacher and the teacher is supported by one member of the research team.

3. Retrospective analysis of what happened in class related to the anticipated classroom activities and how to use this knowledge to design the next problem or to revisit the same problem.

The purpose of the activities on the first day and the last day were different. The first day was devoted to planning and the last day was devoted to the analysis of the week’s activities. Table 1 shows the different activities during the workshop held at the teacher training centre and at the school.

Table 1.
Overview of 5-days Workshop Activities

<table>
<thead>
<tr>
<th>Day</th>
<th>At teacher education college</th>
<th>Activities</th>
<th>at school</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>• Designing activities</td>
<td></td>
<td>• Carrying out activities</td>
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<tr>
<td></td>
<td>• Making predictions of</td>
<td></td>
<td>• Observing students’ working on activities (group work)</td>
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<td></td>
<td>students’ thinking</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>• Analysing students’ work</td>
<td></td>
<td>• Finalising posters</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>• Carrying out discussions on posters</td>
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<td></td>
<td></td>
<td></td>
<td>• Observing whole dynamics of whole class discussion</td>
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<tr>
<td>3</td>
<td>• Analysing students’ work</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>• Designing follow up problem</td>
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<tr>
<td></td>
<td>• Predicting students’ thinking</td>
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<tr>
<td>4</td>
<td>• Analysing students’ work and whole class discussions</td>
<td></td>
<td>• Carrying out follow-up problem</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Observing whole dynamics of whole class discussion</td>
</tr>
<tr>
<td>5</td>
<td>• Reflecting on the whole week</td>
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</tbody>
</table>
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Results and Discussion

In discussing the results, we will start with the description of activities involved in each phase of the design research, starting with the design phase, followed by the teaching experiment phase and the retrospective analysis involved in the daily mini cycles. It should be noted that in the daily mini cycles interplay between what happened in the classroom during the teaching experiment and analysis of students’ behaviour and their thinking could not be exclusively separated. Analysis of the teacher’s questions and
students’ discussions will follow the sequences of activities given in Table 1.

**Design Phase**

On Day 1, activities were carried out in the teacher training centre involving the research team and the resident teacher. The rice problem (see Figure 1) was introduced to the whole group. Central to the discussion was the idea to make this story ‘alive’ and meaningful to students. Based on the discussion, it was decided that the teacher would tell the story as her own personal problem and ask for students’ help to solve her problem. Discussion also explored predictions of students’ responses and strategies, including what possible mistakes students would make in solving this problem. The sequence of activities is shown in Figure 2.

**Teaching Experiment Phase**

The teacher, Ms. Hana introduced the problem to her Grade 5 students in an engaging way. The students worked in groups of four and there were five groups in this Grade 5 class. Students worked in small groups to discuss and solve the rice problem. The lesson was observed by the research team consisting of seven people. Despite the teacher’s effort to make the context meaningful and engaging for the students, the students’ initial attempts in solving the problem suggested their tendency to use procedural algorithms such as the division algorithm and the multiplication of fractions as documented in Figure 2. This suggested that students had ignored the context presented in the problem and focussed their attention on the numbers. This explained the students’ attempts to apply multiplication instead of division in their strategy. One group employed the repeated subtraction strategy of \( \frac{82}{4} - \frac{3}{4} = \frac{79}{4} \) which indicated a better understanding of the problem. However, as this strategy involved lengthy steps to get to the answer, a calculation mistake was fairly possible as can be seen in Figure 3.
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During the group discussion, the teacher walked around the classroom, observed the group discussions and posed questions to clarify students’ ideas. Observing the discussions in the small groups, the teacher realised that the majority of her students did not attend to the contexts in solving the problem in a meaningful way. Instead, they tried to find and apply any operation on fractions that they were familiar with. Their attempts to multiply the two numbers, i.e., $\frac{3}{4} \times 25$ instead of making sense of the problem as a division problem indicated that the context, as it was understood by the children, did not help them to make sense of the situation (see Figure 3). Furthermore, the teacher noticed that some students were not able to explain what $\frac{3}{4}$ means. Figure 4a shows three open circles on the top and four closed circles on the bottom which was the representation one student had made of $\frac{3}{4}$. Apparently, this drawing shows that $\frac{3}{4}$ was perceived as a fragmented notion of collections of three things over four things.

To help students realise what they were doing, the teacher re-introduced the problem; this time by emphasising the context and asking the student to represent $\frac{3}{4}$ kilos of rice using drawings of one-kilogram boxes of rice. One student drew the representation of $\frac{3}{4}$ as shown in Figure 4b. In contrast, this representation showed an understanding of $\frac{3}{4}$ using a part-of-a-whole interpretation of fractions, that is, three parts of one whole consisting of four equal parts.

Figure 3. Samples of students’ initial attempts to solve the rice problem.
Figure 4. Representations of three fourths.

The teacher capitalised on this new representation of $\frac{3}{4}$ to encourage the rest of the class to re-think and make sense of this situation. Ms Hana’s remark and questions were critical in moving the student’s away from their initial attempts of using formal but meaningless algorithms. The scripts below presented some of conversations during the whole class discussion:

Teacher : If one day, I need $\frac{3}{4}$ kilogram of rice, now if I have 3 kilograms of rice, how many days do you think this rice will be enough?
Jojo : 4 days
Teacher : Jojo, you say it is enough for 4 days, could you explain why?
Jojo : Can I draw other rectangles?
Teacher : Yes, sure

Jojo drew another three equivalent rectangles as shown in Figure 5 and continued:

Jojo : In each of the rectangles, three of these (referring to $\frac{3}{4}$ of parts in each rectangle) which means they are enough for 3 days but in each of the rectangles, there was one part left so three of them would be enough for 1 more day so in total the rice would be enough for 4 days.

Teacher : Who has understood Jojo’s explanation? Sari, could you explain what Jojo has said before?
Sari: Each three of these (referring to a rectangular representation of \( \frac{3}{4} \)) are for one day so 3 of these are for 3 days. The remaining three of these (1/4 kilos) are combined and they are enough for 1 day so 3 kilos are enough for 4 days.

Teacher: Have you understood this explanation? Anyone would like to ask questions?

![Representations of 3 kilograms of rice.](image)

**Figure 5.** Representations of 3 kilograms of rice.

Drawing on the students’ idea, Ms Hana then tried to bring students back to the context of the problem by asking the following question, “Now, you said that 3 kilos are enough for 4 days so now how about 25 kilograms? Do you want to try to solve that?” Various strategies were documented following this episode of classroom discussion as shown in Figure 6. It was observed the learning atmosphere was changed as many groups now employed the context and representation of \( \frac{3}{4} \) discussed earlier in the class. Even though the given representation seemed to ‘show’ one strategy in solving the problem, the groups utilised a wide range of strategies. At the end of this lesson, students put their work into posters. These group posters were then presented during class discussion on the following day.
The next day, prior to a poster presentation session, the class started with ten minutes of group work to finalise the posters. The teacher, Ms. Hana, chose one group to present their work in front of the class (see Figure 6). This group was selected because their work had the potential to elicit interesting mathematical discussion on division of fractions. This group constructed their solutions using a rectangular model for fraction $\frac{3}{4}$ from day 2 and noted that 3 kilograms of rice were sufficient for 4 days and came to a conclusion that 25 kilograms of rice would be used up in 33 days with $\frac{1}{3}$ of kilograms left-over. All members of the group were asked to come to the front of the class during the presentation so that they could share the ownership of their work. For the ease of presentation, the group decided to appoint one student as the group speaker to share the strategy. Following the presentation, Ms Hana invited the other students to pose questions to the group. This action allowed the rest of the class to engage in a meaningful construction of their knowledge and to bring their strategies into the classroom discussion. Moreover, Ms. Hana’s action gave room for the group who had presented to justify their strategy, which was accepted as one of the socio-mathematical norm in the classroom. One group questioned the solution of 33 days and instead of $\frac{1}{4}$ as the remaining part of the rectangles represents one fourth of a kilo. The teacher did not tell the students which was the right answer. Instead she again invited the rest of the class to figure out the answer themselves.
A member of research team noticed that in the poster, the students had employed a proportional thinking marked by red colours in the poster (Figure 7). On this poster, the students encircled 8 groups of three boxes or 4 days with red markings. It was brought up to the teacher’s attention that asking about the role of the red markings could serve as an entry point for students to think of another strategy using the ratio table. The poster indicated that the students had started to think in this direction but they had yet to articulate the use of ratio table as part of their strategy. Therefore a question that would ‘guide’ students to move forward in this direction would be needed. Ms. Hana followed this suggestion and asked her students to explain what was the role of red markings in the poster (Figure 7).

![Figure 7. The poster used in the whole-class discussion on Day 3.](image)

The students were encouraged to observe the poster closely by coming to the front of the class. Eight students came to the front and one boy, Jojo, made a remark “yes, yes… I understand now”. When the teacher posed again the question about the role of the red marking, two students explained that it was used to group the amount of rice enough for four days and the red marking helped to calculate the number of days by grouping. When the teacher asked another student to summarise this in writing, Bobi wrote “3 boxes represent the amount of rice enough for 4 days”. Following this remark, the teacher asked other students to comment on this answer and many
students raised their hands. One student explained that “because 3 boxes are for 4 days then if we multiply by 2 then we get 8.” These responses suggested that the student already had an idea about ratio. Therefore to facilitate this move forward to a more formal thinking, Ms. Hana introduced the ratio table to highlight the relationship between ratio and proportion. It is the understanding of this relationship which helped the students to reorganise their ideas about multiples into a ratio table. As can be observed in Figure 7, because the students already had applied some proportional thinking in their strategy, the idea about ratio table was picked up quickly.

On the last day, the students’ started by thinking about the relationship between kilograms and days. The students had encountered a problem as some of them thought that the remainder was $\frac{1}{3}$ while others thought that the remainder was $\frac{1}{4}$. The teacher revisited this discussion and students came to the conclusion for themselves that there are two ways of thinking in solving this problem; 33 days and $\frac{1}{4}$ of kilo remainder, or 33 and $\frac{1}{3}$ days. Clearly, not only the understanding of the relationship was important. The context itself helped the students to understand the differences between the two remainders.

Finally, Ms. Hana introduced a similar problem of dividing 25 kilograms of rice where they cooked $1\frac{1}{4}$ kilo/day. The students’ solution clearly showed various approaches at different levels of formality, including using drawing of contexts, ratio table, and formal solution by multiplication of fractions as documented in Figure 9.
Retrospective Analysis Phase

Based on our observations during the teaching experiment phase, we contend that the teacher’s encouragement for the children to think about various possibilities of solving the problem and not telling her students the right or wrong answers had stimulated students to think for themselves. It was also a lesson for us that even when the context was introduced by the teacher in an interesting way on the first day, it did not automatically imply that students will utilise the context in solving the problem. As shown in our data, the students initially attempted to fit any operation on fractions that they knew. As a result, many of these attempts resulted in meaningless operations and solutions. Our data suggests that it is crucial to encourage students to move away from meaningless operations so that students could develop their thinking. The use of representation for a fraction was one of the crucial teaching points. The teacher’s probing questions which highlight the context in the problem was another critical step in helping students comprehend the problem in a more meaningful way in contrast to their initial solutions.

Contrary to the classical set up, the classroom arrangement wherein students worked in small groups allowed more space for students to explore the problem and discuss their strategies in a constructive way. The set up
also permitted the teachers to observe students’ various strategies and thinking including their struggles as they discussed their solutions and justified their reasoning. In retrospect, we observed a positive impact of asking probing question on students’ reasoning, and encouraging students to decide for themselves whether an answer is right or wrong and whether their argumentation is acceptable.

The research team played a key role in pointing out key mathematical points which allow students to move forward to a higher mathematical level. In our study, the research team assisted the teacher in selecting the poster to be discussed in the classroom. We discussed with the teacher its possibility as an entry point for discussing another strategy using the ratio table. Initially the research team was a bit cautious about selecting only one poster for the classroom discussion as it would make the other students disappointed. However, we observed that Ms. Hana invited students to understand the strategies used by the other groups and to explain their understanding in their own words. As a result, we noticed that the classroom engaged in rich mathematical discussions.

At the end of the 5-day workshop, the teacher, Ms. Hana, made a remark about that the powerful role of the contexts and probing questions in advancing students’ thinking. She acknowledged in her reflection that this experience had expanded her insight about realistic mathematics education. Previously her impression of realistic mathematics lesson was mainly characterised by the use of concrete materials. From this workshop she learnt about the power of contextual problems as a starting point to build students’ thinking and about ways to advance students’ thinking with probing questions.

**Conclusions and Implications**

The data in this study shows that contexts play a powerful role in bringing out rich mathematical thinking and discussion amongst students when they are meaningful and comprehensible. Clearly, the teacher plays a central role in eliciting the discussion that allows students’ thinking to grow. Meaningful contexts combined with teacher’s probing questions - highlighting big mathematical ideas - allow students to attain various approaches at different levels of formal mathematics. However, we do not imply that this is a common practice in Indonesian classrooms. Building classroom norms where students are responsible to justify their strategies and solutions as
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well as to ask for clarifications from others instead of relying on the teacher to justify their thinking is critical. Cultivating this set of practice in learning mathematics requires a change a paradigm from the teachers and students.

During the course of this design research study, the teacher was supported by the team of teacher educators and fellow teachers on a day-to-day basis. The collaborative effort between the teacher educators and teachers in carrying out this classroom practice was critical in creating an environment that support the learning of mathematics. It is encouraging to learn that during this short design experiment, almost all students in this classroom were able to understand and use contexts to come up with various approaches at different levels of formal mathematics.

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