Improving Mathematical Thinking through Assessment

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National tests, an influential assessment practice in many countries in East Asia, are often blamed as obstacles to good instructional practices. In this paper, the positive impact of assessment, national tests in particular, is discussed using the case of Singapore. The first part of the paper includes an analysis of items from the primary grade national test. It is found that there is a significant emphasis on visualization, number sense, communication and metacognition, and an emerging emphasis on patterning. The impact of allowing students to use calculators in the national test is also discussed. The second part of the paper focuses on the impact of this emphasis on classroom instruction, textbook materials and professional development of teachers. This paper aims to show that, instead of being an obstacle to good instructional practices that develop mathematical thinking, large-scale assessment such as national tests can be the catalyst.

Introduction

High-stakes testing is an influential assessment practice in education systems around the East Asian region. The term tests is used to refer to standardised tests where they are given under uniform conditions and students taking the tests are asked to respond to the same or almost the same set of test items. High-stakes tests have serious consequences. Students may be required to pass a test to be promoted to the next grade. Their performance on the tests may determine their subsequent course of study. They may be awarded a certificate if they pass a test, or denied one if they fail the test. Such certificates may be essential in obtaining a place in a university and are useful when applying for jobs. In Singapore, where there is high-stakes testing in Grade 10, grade eight teachers were found to place a much greater

Key words: National test; Assessment; Mathematical thinking; Problem solving
emphasis on classroom tests as a tool to monitor students’ progress compared to their own professional judgment (Mullis, Martin & Foy, 2008). The Trends in Mathematics and Science Study (TIMSS) 2007 found similar trends in other East Asian countries. A lot more students were taught by mathematics teachers who reported that they were more likely to use classroom tests than their own professional judgment to monitor students’ learning.

Table 1

<table>
<thead>
<tr>
<th>Country</th>
<th>Percentage of Grade 8 students whom teachers placed major emphasis on this source to monitor students’ progress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Teachers’ Own Professional Judgment</td>
</tr>
<tr>
<td>Chinese Taipei</td>
<td>17</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>34</td>
</tr>
<tr>
<td>Japan</td>
<td>7</td>
</tr>
<tr>
<td>South Korea</td>
<td>19</td>
</tr>
<tr>
<td>Indonesia</td>
<td>36</td>
</tr>
<tr>
<td>Malaysia</td>
<td>19</td>
</tr>
<tr>
<td>Singapore</td>
<td>23</td>
</tr>
<tr>
<td>Thailand</td>
<td>6</td>
</tr>
<tr>
<td>International Average</td>
<td>45</td>
</tr>
</tbody>
</table>

In Singapore, students in Grades 6, 10 and 12 sit for national tests. In Grade 6, students are tested in English, mathematics, science and mother tongue languages. Despite an increased emphasis on a more holistic assessment to enhance learning (Ministry of Education Singapore, 2009), the national test remains an important feature in the education system (Tan, Chow & Goh, 2008). Students’ results in the Primary School Leaving Examination (PSLE) determine the secondary schools students go to and the course they study in subsequent grades. In Grades 10 and 12, students receive certificates for passing the national test. The certificates received for passing the Grade 12 national test, the General Certificate of Education Advanced Level (GCE A Level), are also used to gain places in universities.

Wilson (2007), in a review on research in high-stakes testing in mathematics, reported on the effects of such high-stakes testing. High-stakes
testing has often been blamed for poor instructional practices. A substantial majority of teachers in a national survey in the US reported that state tests had led them to teach in ways that contradicted their ideas of sound instructional practices (Pedulla, Abrams, Madaus, Russell, Ramos, & Miao, 2003). However, it was found that the test format of high-stakes test has some positive influence on instruction. In the same national survey, far more teachers in states with high-stakes tests gave classroom tests with formats similar to the ones in the high-stakes tests than in states without high-stakes tests (Pedulla et al., 2003). When tests use more open-ended items that require writing, teachers place more emphasis on writing in mathematics classes (Taylor, Shepard, Kinner, & Rosenthal, 2003). In this paper, we study the case of such high-stakes mathematics tests in Singapore, focusing on the compulsory national test taken by students at the end of primary school. This national test is referred to as the Primary School Leaving Examination (PSLE). We argue that national test can be a tool that leads to instructional practices that contributes to mathematical literacy.

Primary School Leaving Examination (PSLE) Mathematics

In the first part of the paper, the findings from an analysis of test items in the Primary Six national test are presented. In this high-stake national test, there are 15 multiple-choice items contributing 20% of the total score, 20 constructed-response items that require answers with little or no explanation contributing 30% of the total score, and 13 constructed-response items that require extended response contributing to 50% of the total score.

Released items from the national test in the last five years (2004 - 2008) were selected for analysis. In 2009, the format of the test was modified to include the use of calculators in problem-solving items. According to the examination board, there was no change in the item types, the number of items for each type or the level of difficulty of the items as a result of the introduction of the calculator (Ministry of Education Singapore, 2007). For the purpose of the present study, only items from tests that did not allow the use of calculator were used. Over the period of five years, 191 items out of 240 (80%) were released. The analysis proceeded in two stages. In the first stage, each item was coded as basic-skill items, direct-application item or challenging items. Basic-skill items are computation items or those that require basic knowledge. In the second stage, the challenging items were qualitatively analysed to determine the emphasis of such items. Figure 1 shows examples of items coded as basic-skill items.
Figure 1. Examples of basic-skill items.

Direct-application items are those that involve one or two steps of direct application of skills or concepts. Figure 2 shows some examples of items coded as direct-application items.

Figure 2. Examples of direct-application items.

Challenging items are items that possess the quality of complexity or novelty. Novel items are items not found in previous years’ tests. Complex items require students to see an element that is not immediately obvious and often requires multi-step thinking. For example, the item in Figure 3 requires students to use the fact that Plank P is 240 cm in order to find the
distance between two consecutive holes on Plank P. Students then need to find the length $AB$ and conclude that $CD$ is of the same length using a property of rectangles. Next, students have to find the distance between consecutive holes on Plank Q and deduce the length of Plank Q. This item requires multi-step thinking and is coded as a challenging item. Metacognition is probably essential in organising the many steps needed to complete the task. The item also requires the ability to visualise. Several examples of challenging items are given in the next few figures.

Four planks P, Q, R and S are nailed together to make a frame as shown below.

Plank P has 7 holes which divide it into 8 equal parts.
Plank Q has 5 holes which divide it into 6 equal parts.
In the frame, the holes A, B, C and D are four corners of a rectangle.

Plank P is 240 cm long.
What is the total length of Plank P and Plank Q?
(1) 600 cm (2) 510 cm (3) 492 cm (4) 450 cm

(SEAB, 2009, p.9)

Figure 3. An example of challenging items.
Another item that was coded as a challenging item (Figure 4) requires students to see the relationship between the before and after ratio, which is not obvious. Metacognition is necessary and because this is a constructed-response item, students are required to communicate a response that is fairly extended.

At first, the ratio of Shanti’s savings to Roy’s savings was 5 : 4. After each of them donated $60 to charity, the ratio of Shanti’s savings to Roy’s savings became 13 : 10. What was Shanti’s savings at first?

(SEAB, 2009, p.53)

Figure 4. An example of challenging items.

Another item that was also coded as challenging item is shown in Figure 5. Students were required to understand the given text and use information appropriately to deduce that there were $27 \times 5$ or 135 books on the three shelves which were removed. Thus, each shelf has $135 \div 3$ or 45 books at first. Although the computation is straight forward, the generation of the two number sentences is not. The calculation in this solution typifies the numbers used in the items when calculators are not allowed. The computations are not tedious and amendable to mental strategies. For example, students can easily half the value of $27 \times 10$ to obtain the value of $27 \times 5$. Similarly, 135 can be seen as $120 + 15$ and finding the value of $135 \div 3$ mentally is straight-forward. It was found that the difficulty of the items coded as challenging was not from tedious computation but from other cognitive and metacognitive demands.

At first, the books in a library were placed on 30 shelves with an equal number of books on each shelf. 3 shelves were removed and the books on these shelves were placed on the remaining 27 shelves. Because of this, the number of books on each shelf increased by 5. What was the number of books on each shelf at first?

(SEAB, 2009, p.20)

Figure 5. An example of challenging items.
Emphasis of Mathematics Test Items in Singapore Primary School Leaving Examination

The number of items coded as basic-skill, direct-application and challenging items are shown in Table 2.

Table 2

<table>
<thead>
<tr>
<th>Main Topic</th>
<th>Basic-Skill Items</th>
<th>Direct-Application Items</th>
<th>Challenging Items</th>
<th>Total Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers</td>
<td>42</td>
<td>11</td>
<td>19</td>
<td>72</td>
</tr>
<tr>
<td>Measurements</td>
<td>7</td>
<td>19</td>
<td>14</td>
<td>40</td>
</tr>
<tr>
<td>Rate</td>
<td>1</td>
<td>8</td>
<td>10</td>
<td>19</td>
</tr>
<tr>
<td>Ratio &amp; Percentage</td>
<td>2</td>
<td>7</td>
<td>10</td>
<td>19</td>
</tr>
<tr>
<td>Algebra</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Geometry</td>
<td>11</td>
<td>12</td>
<td>11</td>
<td>34</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>63</strong></td>
<td><strong>64</strong></td>
<td><strong>64</strong></td>
<td><strong>191</strong></td>
</tr>
</tbody>
</table>

The number of items coded as basic-skill, direct-application and challenging items are almost evenly distributed. However, given that basic-skill items are given lower credit per item, it can be seen that more emphasis is given to items coded as challenging items. It can be thus concluded that there is a significant emphasis on problem solving and the ability to handle complexity and novelty in the primary school national test items in Singapore.

The qualitative analysis of items coded as challenging also show that these items tend to demand metacognition and communication. Almost always, the situations are complex and require students to monitor and control their thinking process as they proceed. Also, given that the items are constructed-response items that require students to show their solution methods to gain credit, there is an emphasis on communication. Many of the items are also presented in verbal form and hence students must be fairly competent in the language to even have a chance to attempt them.

The qualitative analysis also revealed that many of the items coded as challenging require visualisation ability. In some of these items, students need to comprehend fairly complex diagrams (for e.g., see Figure 3). In others,
they need to translate verbal information into visual forms. In all these items
the difficulty is not from tedious computation but from other obstacles such
as complexity and novelty. The computation can be done using mental
strategies, for items where calculators are not allowed.

The qualitative analysis also showed that there is an emerging trend for
items coded as challenging to include patterning. Figure 6 shows one such
example.

Siti started saving some money on Monday. On each day from
Tuesday to Friday, she saved 20 cents more than the amount she saved
the day before. She saved a total of $6 from Monday to Friday. How
much money did she save on Monday?

(SEAB, 2009, p. 41)

Figure 6. An item that includes patterning.

Discussion
National tests are sometimes blamed for poor classroom instruction because
the common rhetoric is that teachers teach to the test. However, an analysis
of the case of Singapore primary school national test, which is high-stakes,
suggests that there is a way national test can result in good pedagogical
practices that promote mathematical thinking. In the case of Singapore, an
analysis of released items from the national test shows that there is an
emphasis on challenging skills and a de-emphasis on tedious computation
skills. Mental computation skills using ‘friendly’ numbers are encouraged.

This finding is further supported by the introduction of the use of
calculator in the primary school national test from 2009 onwards. In one
part of the examination comprising of 10 short-answer items and 13 long-
answer items and contributing to 60% of the total score of the test, students
are allowed to use calculators. This policy is consistent with the emphasis
on higher-order skills such as visualisation.

The use of calculator allows the inclusion of other higher-order
competencies such as patterning, investigating and modelling. Although
these competencies have always been encouraged by the curriculum, they
are difficult to be included in test items when calculators are not used. In
patterning items, students may need to do many repetitive computations
before patterns emerge. In investigative problems, it is also likely that one has to do repetitive computations. In modelling problems based on real-world and authentic data, the computations are often messy and tedious without the use of the calculator.

Thus, the national test has been emphasising competencies which are often described as high-order ones and also form part of mathematical thinking. There seems to be an increased emphasis on these competencies at the expense of more procedural skills. As the national test is high-stakes, schools adjust and refine their instructional programmes and teachers continue to seek new ways via professional development to meet the demands of the national test. This leads to improvement in pedagogical practices and students are exposed to competencies which lead to higher mathematical literacy. Data from studies show that many teachers in Singapore used item formats and items that focus on applications. For example, the TIMSS 2007 data showed that 83% of Grade 8 students were taught by teachers who reported that they used only or mostly constructed-response items (Mullis, Martin & Foy, 2008). The international average was 44%. Similarly, 75% of these students were taught by teachers who reported always or almost always using questions involving application of mathematical procedures (Mullis, Martin & Foy, 2008). This supports the point that teachers will teach to the test. If the national test demands competencies which are part of mathematical thinking then teachers will find a way to teach this. If the national test requires students to articulate their methods and show their thinking then teachers will emphasise these. In other words, if the national examinations include high-quality test items, this will influence instructional practice in the positive sense. However, the education system must have structures in place to support the teachers. This has been the case of Singapore schools where professional development has always been emphasised and has recently received increased attention (Ng, 2009).

In the same way, the textbooks follow the trends of the test items. An analysis of the common textbooks used in Singapore shows an emphasis on mental strategies, visualisation and patterning. Multi-step problems and strategies, especially visual ones such as the model method, are common. There is an almost complete absence of multiple-choice items in the textbooks.
In conclusion, national test can bring about good instructional practices if the test items are of a high quality, if there is a support system for professional development to help schools and teachers find innovative ways to help each student maximise his or her potential, and if good instructional materials such as textbooks are readily available.

Note:
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