Journal of Science and Mathematics
Education in Southeast Asia
2013, Vol. 36 No. 1, 23-43

The Design of a Mathematics Problem Using Real-life Context for Young Children

Lu Pien Cheng
National Institute of Education, Singapore

In this paper the teachers' learning journey on designing a mathematics problem that involves a real-life context is reported. This is part of a larger project undertaken in a primary school in Singapore where the teachers were engaged in planning, observing and critiquing mathematics lessons to investigate teaching and learning. These unique features from laboratory class cycles were used to facilitate the design and implementation of mathematics problems involving real-life context. Based on one cycle of a laboratory class of an authentic classroom lesson, the teachers' construction of the mathematics problems embedded in real-life context is reported. The children's solutions to one of the problems are also discussed in this paper.

Keywords: Primary school mathematics; Real-life context; Young children

Introduction

Increased emphasis on tasks that promote applications and connections of mathematics to the real world had been called for by researchers and several reform curriculum frameworks in several countries since the 1990s (Boud & Feletti, 1991; NCTM, 1989, 2000; Romberg, 1992; Schiefele & Csikszentmihalyi, 1995; Streefland, 1991; Trafton, Reys, & Wasman, 2001). In Singapore, the primary school mathematics curriculum has consistently emphasised the application of mathematics to solve real-world problem (MOE, 2001, 2007). The 2001 mathematics curriculum states that "mathematical problem solving includes using and applying mathematics in practical tasks, in real life problems and within mathematics itself" (p. 5). The 2007 mathematics curriculum states that mathematical problem solving involves the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems (MOE, 2007).

Real-Life Context

Beswick (2011) described the use of terms such as *authentic*, *real life* and *situated* to reflect "different degrees of distinctiveness from problem presentations that rely entirely on mathematical symbols" (p. 368). The Realistic Mathematics Education (RME) developed by the Freudenthal Institute is also known as 'real-world mathematics education' (Van den Heuvel-Panhuizen, 2000, p. 4). In RME, mathematics is viewed as a human activity which connects mathematics to reality. *Reality* here refers to mathematics that is relevant to everyday situations and problem situations that are real in students' mind. de Lange (1992) argued that mathematics should never be presented to the students as a ready-made product when it is viewed as a human activity. Instead, students should re-invent mathematics. According to Freudenthal (1973, 1991), opportunities should be given to children to reinvent mathematics by mathematising (cited in Gravemeijer, 2004). In this paper, real-life context problems refer to problems embedded in real life situations that have no ready-made algorithm. The words *problem* and *task* are also used interchangeably. The term *context* in this study refers to the notion of setting and situation (Roth, 1996). "Social, physical, historical, spatial, and temporal aspects" (Roth, 1996, p. 491) are considered when referring to situations while the "various physical sites of human activities" (p. 491) are considered when referring to setting. In this paper, we define real-life context to include situations that refer to everyday activities (Stylianides, 2005).

Mathematical tasks embedded in real-life contexts are abundant in reform-based mathematics textbooks (Stylianides, 2005; Trafton, Reys, & Wasman, 2001). One key feature in tasks embedded in well-designed real-life contexts is its ability to motivate and engage students in learning mathematics (Schiefele & Csikszentmihalyi, 1995; Trafton et al., 2001). However, there are few occasions in our everyday life where "algebracising the situation" (Filloy & Sutherland, 1996, p. 152) is useful. It is often challenging for teachers to implement cognitively high-level real-life tasks such that the mathematics involved in the tasks are not overshadowed by the motivational features of such tasks (Stylianides & Stylianides, 2008). Thus, teacher preparation and professional development programmes play a critical role in equipping teachers with the necessary mathematical and pedagogical knowledge to help teachers understand and appreciate the mathematical affordances of tasks.

Studies involving real-life context in mathematics education can be found in professional development, teacher education programmes and in the mathematics classrooms. In a study by Ben-Chiam, Keret, and Ilany (2007), authentic proportional reasoning tasks were used to assist pre-service elementary mathematics teachers to acquire and understand ratio and proportion topics. Many of the pre-service teachers were able to incorporate the authentic task successfully and they were also more encouraged to implement similar tasks in their own classrooms on a regular basis, when they started their own teaching. The study by Gainsburg (2008) with secondary mathematics teachers revealed that teachers have different priorities. The teachers viewed the imparting of mathematical concepts and skills as more important to developing students' ability and disposition to solve real problems. The study suggested that professional development programmes should demonstrate how real-world connections can build mathematical mastery through the real-world activities so that these activities can be used more often in the mathematics classroom. In a professional development programme to help elementary school teachers connect in-school and out-of-school mathematical understandings using problems embedded in real-life context, Taylor (2012) suggested that specific support be made to address lesson design that was built on students' informal understandings.

Solving problems embedded in a real-world context requires to some extent students' informal understanding of the context. This paper delves into the design of mathematics problems in real-world context and attempts to demonstrate how real-world connections can build mathematical mastery in the mathematics classroom. By examining these two aspects, this paper seeks to provide some insight into the mathematical and pedagogical knowledge required of such problems so that teachers can optimise the richness of mathematics afforded by these tasks.

Task Design Principles

In designing the problems in this study involving real-life context, the framework of purpose and utility (Ainley, Pratt, & Hansen, 2006) were used. The utility of mathematical ideas was ensured in the selection of the tasks by choosing tasks based on the usefulness of the mathematical ideas to the children. For example, in this study, the children appreciate and develop the skills of different ways of making up money in dollars and cents through the *Restaurant* problem in Table 3.According to Ainley et al. (2006), the teacher needs to imagine the trajectory of a pupil's thinking during an activity from the mathematical and learner-centered perspective to ensure that the tasks offer both purpose and utility. This makes designing such tasks challenging.

Laboratory Class Cycle

In this study, the laboratory class was viewed as a teaching cycle which consisted of three unique features from lesson study where teachers were supported in their learning through preparation, observation and analysis of mathematics lessons. The role of the author was to facilitate and support the teachers' learning as they were engaged in the laboratory class cycle. The preparation phase occurred before the actual mathematics lesson, while the analysis was done after the lesson. The observation occurred during the mathematics lesson. The design of the professional learning tasks in this study considered the teachers' work and the nature of activities in the cycle (Smith, 2001). One complete laboratory class cycle consisted of six consecutive meetings over six weeks. Four of the meetings were used to plan the research lesson. One meeting was used for the teachers to observe the research lesson and another meeting in the cycle was used to critique the observed research lesson. The critique included discussion of children's solutions to the mathematical problems and suggestions to reinforce any teaching ideas that emerged during the laboratory class cycle.

This Study

Purpose of Study

The study reported in this paper forms part of a larger study that involved teachers in laboratory cycles over a period of two years. In this study, the teachers' professional development journey to design a mathematics problem that helped children unpack the mathematics embedded in a real-life context problem was examined. Specifically, the research questions that guide this paper are as follows:

1. What were the features and challenges in the process of designing a mathematics problem in a real-life context?
2. What were the children's solutions to the mathematics problem embedded in real-life context?

Table 1 describes the duration and purpose of the meetings. The lesson was conducted by one of the teachers during the fifth meeting.

Table 1
Meeting and Data collection Schedule for Research School

Meeting	Purpose of Meeting	Duration
1 - 4	Discuss and plan a Primary 2 mathematics task and lesson to develop children's decision making skills through real-world problems in mathematics.	4 hours
5	Carry out the lesson	1 hour
6	Critique of lesson	1 hour

Participants

A total of five teachers teaching the lower primary participated in the study. The participants came from the same school and had varying years of teaching experience. Mary, one of the participants in the study, taught the research lesson to her own class. Mary's class was the better of two classes in the Primary 2 cohort in the school. Prior to the observed lesson, the children were arranged into mixed-ability groups of three children in a group. This grouping was chosen so that scaffolding can be done more appropriately for the observed lesson. Further, it was noted that the children in this class were already familiar with the two-step word problems presented in a simple sentence with ready-made algorithm using real-life context. An example of a two-step word problem that the children were familiar with is as follows: Ali is 80 cm tall. Bala is 10 cm taller than Ali. Caili is 5 cm taller than Bala. How tall is Caili?

Research Design

The study reported in this paper used the account of practice methodology by Tzur, Simon, Heinz, and Kinzel (2002) which follows a case study methodology where researchers unfold what the teacher does and the perspectives that underlie the teacher's practice. The account of practice methodology was chosen because a major goal of this study was to explicate the teachers' experience and practice using the laboratory class cycle to design a mathematics lesson. Weekly meetings were tape recorded so as to capture the teachers' conversations about classroom practice, task design, and their process of constructing a lesson plan. Artifacts from the weekly meetings and the researcher's field notes of children's work and the weekly meetings formed part of the data set. Using these data sets, an account was generated through intensive line-by-line analysis of transcripts while listening to the audio recordings as well as reflecting on the researcher's field notes and the children's work.

Analysis of Data

Sections of the data that were able to illuminate the teachers' decisions when designing the mathematics problems were identified. The kinds of decisions that teachers made were considered because they offer some insights into the construction of mathematics problems embedded in real-life context that foster the mastery of mathematical skills for young children. Sections that revealed the teachers' analysis of the children's solutions to the *Restaurant* problem were also reported as they provided illuminations about the ways the teachers develop a deeper understanding of young children's thinking of the *Restaurant* problem. By understanding how young children solve the *Restaurant* problem, the teachers sought to design and use more of such problems to promote children' learning of mathematics.

Findings

Analyses of the teachers' conversations during the laboratory class revealed aspects that the teachers considered as important in designing the problems. First, we consider the teachers' collective decision-making about the mathematics problems using real-life contexts and ways in which the real-world problem should be introduced to the children. We then discuss the teachers' analyses of children's mathematical work. In doing so, insights into the specific ways in which the design of mathematics lesson may be revealed.

Designing the Task

The topic on money was chosen by the team to situate the problem in a real-life context. The lesson seeks to reinforce the children's ability to show the different ways of making up a single currency note with the different denominations in currency and coins, and to show the different ways of making up money in dollars only, cents only or in dollars and cents, besides developing young children' decision making skills through the task. The *Restaurant* problem which was adapted from a problem in the primary school mathematics textbook was designed so that the children would be able to encounter and apply the selected mathematical ideas.

The Textbook problem: Use some of these coins (50cents, 20cents, 10 cents and 5 cents) to make the given amounts of 90 cents, 75 cents, $1.15.

Restaurant problem: *You are going out for lunch at a restaurant with your group. Use the menu to select what you want to eat. The cost of the meal must be less than and as close to $40 as possible. How can we figure out how and what to choose for lunch?*

At first the teachers felt that the *Restaurant* problem was too open-ended for the targeted Primary 2 children and were uncomfortable with the idea of implementing the task. They felt that the children had only been previously taught to solve problems by identifying the 'given' and the 'to find' in the mathematics problem and using certain procedures to arrive at the answer which can be found at the back of their textbooks. To match the task to the perceived children's mathematical ability, the teachers recommended a pre-task to familiarise the children with the open-ended *Restaurant* problem. Subsequently, the teachers chose the familiar context of the school bookshop to construct the pre-task which was designed to include more structured instructions.

In designing the worksheet for the tasks, the teachers paid great attention to the 2007 mathematics curriculum and the children's pre-requisite knowledge. The numerical values in the pre-task were kept small so that mental calculation strategies (such as making 100 using number bonds e.g. 20 and 80 makes 100) can be used to add money. This was done to help the children focus on the development of the process skills (e.g. comparing the cost of the items and deciding which items to choose so that the condition of the total amount to be spent is satisfied) required for the *Restaurant* problem. Figure 1 shows the *school bookshop* pre-task.

Once the children were familiar with the pre task, the initial *Restaurant* problem was then posed to the class. Some changes were made to the initial *Restaurant* problem. The teachers added in two more conditions to the problem. The total amount to be spent was reduced from $40 to $30 so that the problem would be more manageable for the young children. The cost of the items in the menu was then purposefully chosen with the aim of helping children reinforce the following skills:

(1) making up $1 with two 50 cents coins

(2) making up a single note with the different notes (e.g. $12 and $8 makes $20)

The teachers reasoned that once the children were able to group two 50 cents coins into $1, they would be able to combine this $1 with other notes to make another single note. The instructions in Figure 2 were then given to the children.

<table>
<tr><td rowspan="9">You have to spend $2 at the school bookshop during recess. You can buy only one of each item. You must spend exactly $2.

Eraser 10¢ Clip 10 ¢
Pencil 20 ¢ Pencil sharpener 20¢
Ruler 20¢ Highlighter 30¢
Scissors 30¢ Colour pencil 80¢
Glue 50¢</td><td colspan="3">Record your selections in the table. Find the total cost.</td></tr>
<tr><td></td><td>Item</td><td>Cost</td></tr>
<tr><td>1.</td><td></td><td></td></tr>
<tr><td>2.</td><td></td><td></td></tr>
<tr><td>3.</td><td></td><td></td></tr>
<tr><td>4.</td><td></td><td></td></tr>
<tr><td>5.</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td>Total Cost</td><td></td></tr>
</table>

Figure 1. Pre task – School bookshop.

Lunch at Restaurant

You are going out for lunch at *Rose Restaurant* with the friends in your group. Use the menu to select what you want to eat.

Conditions:

1. Each person must have a drink.
2. Dessert is optional.
3. The total cost of the lunch must be **less than and as close to $30 as possible.**

Use the order form on the next page to write in your choices.
(*GST and service charge waived)*

Figure 2. Restaurant problem.

In the subsequent activity, the children were then assigned into groups of three to record and complete the *Restaurant* problem as shown in Table 2.

Table 2
Record Sheet / Order Form

	Name of Friend	Item	Cost
1.			
2.			
3.			
4.			
5.			
	Total Cost		

A sample of the accompanying menu in Table 3 was given to each group to facilitate the process of completing the task.

Table 3
Sample of accompanying menu for each group of children

Menu		
• Bacon and egg muffin 1 for $3.00 • Cheese burger and french fries 1 set for $5.50 • Fish and chips 1 plate for $6.00 • French fries 1 plate for $3 • Fried chicken wings 1 basket of 12 for $18 • Prawn Noodle 1 bowl for $5.00 • Spring roll 1 plate of 4 for $6.00	Drinks (no refill) • Orange juice 1 cup for $2.50 • Ribena juice 1 cup for $1.50	• Ice cream cones 3 for $4.50 • Ice cream 1 for $1.50 • Triple-scoop ice cream for $3 • Mini chocolates 1 box of 16 for $4.00 • Chocolate cake 1 slice for $3.00

Table 4 compares the *School Bookshop* pre-task to the *Restaurant* problem with respect to the skills that the children were expected to transfer from the pre-task to the actual *Restaurant* problem.

Table 4
Comparison of the School Bookshop and Restaurant Problems

Pre-task (*School Bookshop* Problem)	Actual Problem (*Restaurant* Problem)
Purpose of Pre-task: Develop children's competencies in identifying any assumptions, the 'given' and the 'to find' in the first task by teacher modelling to the children how the assumptions were drawn from the problem situation. ***Assumptions*** • Can only buy items stated in the list • Assume the bookshop in the problem is a pretend bookshop. ***Given*** • $2 to spend • Buy each item once only • Spend exactly $2 ***Find*** • Items that you are able to buy using exactly $2	***Purpose of Task:*** Children identify the following 'given', 'to find' and assumptions in the second task. There are more conditions to satisfy in the actual task than the pre-task. ***Assumptions*** • Can buy each item from the menu more than once • Assume the restaurant in the problem is a pretend restaurant • Items can be shared among group members ***Given*** • $30 to spend as a group • Each person must have a drink. • Dessert is optional. • The total cost of the lunch must be less than and as close to $30 as possible. • GST and service charge waived ***Find*** • Items that you are able to buy using exactly $30 or an amount close to $30
Mathematical reasoning skills to be reinforced through solving the pre-task: • Show the different ways of making up a single note ($2) with the different notes and coins.	***Mathematical reasoning skills to be reinforced and developed through solving the pre-task:*** • Show the different ways of making up $30 in dollars and cents. • Making up a dollar using two 50 cents.

The *Restaurant* problem was thus designed to be semi open-ended. Only some assumptions were clearly made known to the problem solver. It was not explicitly stated whether the main dish was a standard item or an optional item. This semi open-ended problem allowed the groups more opportunities to question the assumptions made, and about the known and unknown quantities.

One of the ways to extend the *Restaurant* problem was to make the problem more open-ended which could be done by having the children pose their own problems using the information on the menu. This would provide the opportunity for the children to make their own assumptions for the problem and to recognise the quantities that influence the problem situation. For example, the children may imagine that they were having lunch with their family and they would have to decide how many family members were present for lunch, and the food requirement of each family member. However, such an open-ended problem can perhaps be used at a later stage when children possess a better knowledge of the social norms related to the context. A lesson plan was then designed using the ideas generated. A copy of the lesson plan is found in Appendix A.

Reflecting on the decisions made in the design of the problems in this study, implications about real-life context open-ended tasks were drawn. The following stages appeared to be useful when introducing such problems to young children:

Stage 1 - Pre-task to familiarise the children with the competencies to solve the actual problem related to real-world context,

Stage 2 - Actual task with the competencies in stage 1, and

Stage 3 - Extension of the actual task.

Synthesis of Children's Solutions to the Restaurant Problem

During the whole group critique, the teachers began the discussion by focusing on the children's solution strategies. They examined the children's responses and discussed how the observed planned lesson helped the children build mathematical mastery for the real-world activities. Using data from the critique and the researcher's field notes, the children's responses were categorized into the following types:

Type 1

First, the children would choose any food, drinks and dessert from the menu according to their preferences. Next, they added up the total cost before fitting in the given conditions in the question - they checked the total cost to make sure that it is less than $30 before ensuring that every group member had a drink. During the calculation,

a) if the total cost exceeded $30, the children would remove either the food or dessert items rather than to replace any food items. Since drink is a compulsory item, the children decided not to remove any drinks that they had chosen. The total cost was re-calculated each time items were removed.

b) if the total cost fell below $30, most of the children started adding items, rather than replacing items. The total cost was re-calculated each time items were added.

Type 2

The children would choose their food and drinks (thus meeting one required condition) first. If there was any remaining money, they proceeded to choose the dessert. They then calculated the total cost. During the calculation,

a) Groups that spent more than $30 mostly removed an item from the dessert because it was an optional item. The total cost was re-calculated each time items were removed.

b) Groups that spent less than or more than $30, were more likely to add/remove items respectively than to replace items. The total cost was re-calculated each time items were added/removed.

c) Groups that spent less than $30 mostly added another item, usually from the dessert. The total cost was re-calculated each time items were added.

Type 3

One group divided $30 equally among the three group members. Each group member received $10.Team members chose their own food and drinks independently before coming together as a whole group to calculate the total.

The three types of children' responses showed that the *Restaurant* problem provided opportunities for children to consolidate and develop many process skills outlined in the MOE syllabus (e.g. decision making). By learning and consolidating key skills through use, the children were empowered to learn mathematics in a natural context (Ainley, Pratt, & Hansen, 2006). The specific choice of coins with denominations of fifty-cents helped the children to focus and master the specific skill of renaming and regrouping two fifty cents into a dollar.

During the critique, there was a common observation that some children were satisfied with their solution as long as they spent less than $30. These children did not proceed to select other meals so that the amount spent was as close to $30 as possible. This could be due to a lack of understanding on the meaning of the given condition *as close to $30 as possible* for some children. To address this concern, children could perhaps be given the opportunity to examine and discuss this given condition before solving the problem.

While solving the *Restaurant* problem, some of the children were unsure whether the main course was a standard and compulsory item. This confusion arose because drinks were included whereas desserts were optional. Nothing was mentioned about the main course. Also, the children held different beliefs about what a *proper* lunch meant. Some of the children believed and expected lunch to include at least one main course. Side dishes are optional. Some believed desserts and drinks were sufficient for lunch. This result is in accordance to Chan's (2005) report that the children's choices in their group discussion in problems embedded in real-life context often reflected their personal values and beliefs.

All the teachers agreed that the children were highly engaged and motivated to complete the *Restaurant* Problem. Mary, the teacher who taught the lesson, commented that the whole lesson was "interesting to children, [because] the pupils are not challenged adequately by the word exercises and problems in the textbooks (paraphrase). They were excited that they can choose [through the task]" (Mary, weekly meeting 6).

Conclusions and Implications

This paper analysed the teachers' design of mathematics problems using real-life context for Primary 2 children and the teachers' discussion focusing on the children's solution strategies. The analysis was based on the researcher's field notes, teachers' conversations during the planning and critique of the mathematics lesson. Using the analysis of one laboratory class cycle, the results of this study revealed that three major aspects need to be considered when designing mathematics problems to ensure that mathematical mastery is built through real-world connections for young children.

Knowledge of curriculum is one of the aspects. Extra care has to be taken to situate the problem in a context such that the computational skills involved are appropriate and not beyond the grade level. For example, in the *Restaurant* problem, if the teachers had not chosen items with the appropriate denominators for the notes and coins, the calculations involved would be beyond the ability of the Primary 2 children. Great effort needs to be made to help children make connections between the context and mathematical skills. For example, in the *Restaurant* problem, the teachers connected counting money to whole number addition. By doing so, the teacher helped the children organise their mathematical learning.

Knowledge of task design is the second aspect. The children were highly motivated to solve the *Restaurant* problem because the problem offered them opportunities to make decisions and develop life-skills, unlike the standard practice exercises or word problems taken from the textbook. The *Restaurant* problem also offered children abundant opportunities to count. However, from the three types of solutions observed in this study, it was observed that the children were more likely to add or remove items than to replace items in order to spend as near to $30. Replacing items is a more cognitively challenging task because the children needs to decide which item to replace and the item to be replaced with from the menu. This process requires children to self-regulate their own cognitive processes and work within the task constraints that may limit possible solutions (Stein, Smith, Henningsen, & Silver, 2000). We infer that while the *Restaurant* problem motivates children's learning of mathematics, understanding the cognitive demands of the *Restaurant* problem will enable the teachers to unpack more deeply the rich mathematics embedded in the *Restaurant* problem.

Knowledge of children's thinking and their beliefs is the third aspect to consider when developing a mathematics problem embedded in real-life context. Awareness of children's understanding of the context and that their beliefs and values would shape and determine the kind of choices they make in completing those problems is important for developing and designing appropriate mathematics problems that employs real-life context.

In this paper, one case of a group of teachers engaged in a laboratory class cycle was used to examine the design of mathematics problems embedded in real-life context. While aspects of this case contribute to existing research on mathematics learning and teaching through the use of problems in real-life context, more studies are needed to create stronger theories about the design of mathematics problems using real-life context in developing young children's learning.

References

Ainley, J., Pratt, D., & Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal, 32*(1), 23-38.

Ben-Chaim, D., Keret, Y., & Ilany, B. (2007). Designing and implementing authentic investigative proportional reasoning tasks: the impact on pre-service mathematics teachers' content and pedagogical knowledge and attitudes. *Journal of Mathematics Teacher Education, 10*, 333-340.

Beswick, K. (2011). Putting context in context: An examination of the evidence for the benefits of contextualized task. *International Journal of Science and Mathematics Education, 9,* 367-390.

Boud, D., & Feletti, G. (Eds.). (1991). *The challenge of problem-based learning*. New York: St. Martin's Press.

Chan, C. M. E. (2005, August). *Engaging students in open-ended mathematics problem tasks: A sharing on teachers' production and classroom experience (Primary).* Paper presented at the third ICMI-East Asia Regional Conference on Mathematics Education Conference, East China Normal University, Shanghai, China.

de Lange, J. (1992). Higher Order (Un-)Teaching. In I. Wirszup, & R. Streit (Eds.), *Developments in school mathematical education around the world: Proceedings of the third UCSMP international conference on mathematical education* (pp. 49-71). VA.

Filloy, E., & Sutherland, R. (1996).Designing curricula for teaching and learning algebra. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International Handbook of Mathematics Education* (Vol. 1, pp. 139-160). Dordrecht: Kluwer Academic Publishers.

Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht, The Netherlands: Reidel.

Freudenthal, H. (1991). *Revisiting mathematics education*. Dordrecht, The Netherlands: Kluwer Academic.

Gainsburg, J. (2008). Real-worlds connections in secondary mathematics classrooms. *Journal of Mathematics Teacher Education, 11*, 199-219.

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6*(2), 105-128.

Ministry of Education (MOE). (2001). *Mathematics syllabus.* Singapore: Author.

Ministry of Education (MOE). (2007). *Mathematics syllabus.* Singapore: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston: The Council.

National Council of Teachers of Mathematics. (2000). *Principle and standards for school mathematics*. Reston: The Council.

Reed, S. K. (1999). *Word problems: Research and curriculum reform*. Mahwah, NJ: Erlbaum.

Romberg, T. A. (1992). Problematic features of the school curriculum. In P. W. Jackson (Ed.), *Handbook of research on curriculum* (pp. 749-788). New York: Macmillan.

Roth, W.-M. (1996). Where is the context in contextual word problems? Mathematical practices and products in grade 8 students' answers to story sums. *Cognition and Instruction, 14*(4), 487-527.

Schiefele, U., & Csikszentmihalyi, M. (1995). Motivation and ability as factors in mathematics experience and achievement. *Journal for Research in Mathematics Education, 26*, 163-181.

Smith, M. G. (2001). *Practice-based professional development for teachers of mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing Standards-based mathematics instruction*. New York: Teachers College Press.

Streefland, L. (Ed.). (1991). *Realistic mathematics education in the primary school*. Utrecht: Utrecht University Center for Science and Mathematics Education.

Stylianides, G. J. (2005). *Investigating students' opportunities to develop proficiency in reasoning and proving: A curricular perspective* (Doctoral dissertation, University of Michigan, Ann Arbor). Retrieved from http://sunzi.lib.hku.hk/ER/detail/hkul/3841051

Stylianides, A. J., & Stylianides, G. J. (2008).Studying the classroom implementation of tasks: High-level mathematical tasks embedded in 'real-life' contexts. *Teaching and Teacher Education, 24*, 859–875.

Taylor, E. V. (2012). Supporting children's mathematical understanding: Professional development focused on out-of-school practices. *Journal of Mathematics Teacher Education, 15*, 271–291.

Trafton, P. R., Reys, B. J., & Wasman, D. G. (2001). Standards-based mathematics curriculum materials: A phase in search of a definition. *Phi delta Kappan*, *83*(3), 259–264.

Tzur, R., Simon, M. A., Heinz, K., & Kinzel, M. (2001). An account of a teacher's perspective on learning and teaching mathematics: Implications for teacher development. *Journal of Mathematics Teacher Education, 4*, 227-254.

Van den Heuvel-Panhuizen, M. (2000). Mathematics education in the Netherlands: A guided tour. *Freudenthal Institute Cd-rom for ICME9.* Utrecht: Utrecht University.

Author:

Lu Pien Cheng; National Institute of Education, 1 Nanyang Walk Singapore 637616
email: lupien.cheng@nie.edu.sg

Appendix A

Lesson idea for mathematics problems using real-life context

Learning Activities & Teacher Questions	Expected Student Reactions	Teacher Support
Introduction (10-15 mins) You have $2 to spend at the school bookshop during recess. You can buy each item once. You must spend exactly $2. *Teacher questions:* • *If I buy the hole puncher for 80 cents, how much have I left?* • *What can I buy with the remaining $1.20?* • *How would you record what we have discussed in the worksheet?* • *Is that the only choice to spend $2?*	The children may not be able to select the items such that the total cost adds up to $2. Teacher models the computation using whole number addition: Highlighter 30¢ Pencil sharpener 20¢ Pencil 20 ¢ Ruler 20¢ Eraser 10¢ Compute 30 + 20+20+20+10	Teacher demonstrates how the items are chosen such that exactly $2 is spent. Teacher invites children to come up with alternative choices that would satisfy the conditions. Teacher reassures the children that it is alright to get a different solution as the teacher because there are multiple solutions to the problem. Money manipulatives are given to each group.
Launch ▹ You are going out for lunch at a restaurant with your group. ▹ Use the menu to select what you want to eat. Each person must have a drink. Dessert is optional. The total cost of the lunch must be less than and as close to $30 as possible. How can we figure out how and what to choose for lunch?		
Work in groups of 3 (15mins) ▹ Complete the task in your group. ▹ Take note of how you decide what you want from the menu. ▹ Record your selections in the order form. Add the total cost.	Expected student response: ○ Very easy ○ Incorrect calculation ○ Total is more than $30.	<u>Incorrect calculation</u> ○ The teacher double checks and posed scaffolding questions to aid children self-correct the mistake
Peer checking of calculation (5mins) The children exchange their recording sheets to do a quick check of the calculation.		<u>Total is more than $30</u> Teacher takes note of this and discussed with the whole class during whole class discussion.

Lesson idea for mathematics problems using real-life context *(continue)*

Learning Activities & Teacher Questions	Expected Student Reactions	Teacher Support
Whole class discussion ▷ Discuss products of group work. *Teacher questions:* • What are the items your group chose? • Does everyone in the group have a drink? • Does everyone spend less than $30? ▷ Show a piece of work by one group that spent much less than $30. Lead the class to see how the group can spend as close to $30 as possible.	Teacher models the following <u>decision making processes:</u> ○ Everyone orders what he/she wants for drinks first. We chose the drinks first because everyone must have a drink. ○ Next, we choose the main dish because we are hungry and we are at the restaurant for lunch. ○ Add up the total to make sure that we have not overspent. ○ If we overspend here, we have to replace something that cost $... less. ○ If there is money left, we will have a dessert to be shared among the three of us in the group. ○ If there is still some money left, we will have some finger food… <u>Teacher models the computation:</u> Suppose you chose 2 items that cost $5.50 and $1.50. How much have you spent? ○ Count the 50¢ first. ○ 50¢ and 50¢ makes $1 ○ Count the dollars ○ $5 and $1 and $1 makes $7	Teacher asks the class to observe what types of items were selected. ○ What did your group do first? Second? ○ Did you choose the items on your own or together as a group? ○ Which item did you start ordering first, second and third? The drink or the main course or dessert? Why? ○ When do you know you have to stop choosing? ○ Did you change your choice in the middle? ○ Why did you change? ○ If you change your mind, what was the reason for the change? ○ Did you think of replacing it with a cheaper or more expensive item? ○ Which items will you replace? Why? ○ What will you replace such that the amount you spend is as close to $30 as possible? ○ What item will you replace so that there is more variety to the food you have chosen? ○ How many of you ended up with more than $30 when you first calculate your total amount spent? ○ What did you do to reduce the amount that is close to $30. ○ How many of you ended up with less than $30 when you first calculate your total amount spent? ○ What did you do to increase the amount to as close as $30. ○ What do you mean they have more variety of food?

Lesson idea for mathematics problems using real-life context *(continue)*

Learning Activities & Teacher Questions	Expected Student Reactions	Additional Teacher Questions
Compare & Contrast ▹ Pick two groups with amount spent closest to $30 ▹ Analyze which is a better choice.	Yellow group is better because they have more variety of food. They have ice-cream, drinks, noodles, spring rolls, chicken wing, hamburger …..	
Individual worksheet Every child complete their own order form (for groups of 3) using the same menu.	Green, Purple groups spent the same amount $29.50. They are better because their total cost is closest to $30	