



Realistic Mathematics Education

An overview

by

Warabhorn Preechaporn

Teoh Boon Tat

Leong Chee Kin

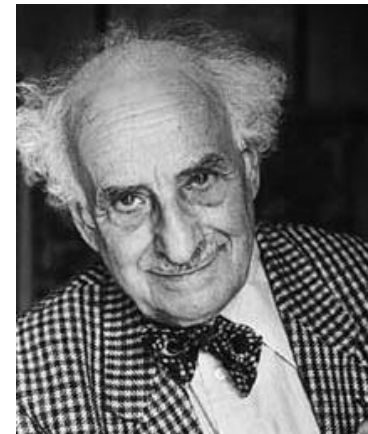
SEAMEO RECSAM, Penang, Malaysia

Outline

- What is RME?
- Philosophy of RME
- Nature and Characteristics of RME
- The Five Tenets of RME
- Hypothetical Learning Trajectory
- Three Heuristic Principles of RME

What is RME?

- Realistic Mathematics Education
- Teaching and learning theory in mathematics education
- Combines the idea:
 - What is mathematics?
 - How mathematics should be taught?
 - How mathematics should be learnt?
- Developed in Freudenthal Institute, University of Utrecht, the Netherlands
- Professor Hans Freudenthal (1905-1990)



Philosophy of RME

Hans Freudenthal believes mathematics must be seen as a human activity of “mathematizing”. Children should not be viewed as passive receivers of ready-made mathematics.

Children should be given opportunity to reinvent mathematics under the guidance of adults (Gravemeijer, 1994).

The reinvention process is developed through exploration of real world problems (de Lange, 1995).

Meaning

Mathematics: knowledge, study, learning includes the study of such topics as quantity (number theory), structure (algebra), space (geometry), and change (mathematical analysis).

Mathematize: to describe in terms of a mathematical equation

Mathematization of a problem or area of study consists of applying mathematical ideas to that problem or field so as to think more precisely or clearly about things.

For example

We can imagine a pile of nails that looks two or three times larger than another pile of nails. Assuming that the nails are the same in both piles and that they are packed about the same we can readily conclude that there are more nails in the bigger pile.

Of course “**bigger**” and “**lesser**” are also mathematical ideas but the counting of the nails in each pile can further **mathematize** the problem and we can finally conclude exactly how many more nails are in one pile than the other.

Nature and Characteristics of RME

- Mathematics instruction emphasizes on meaningful learning: “making sense”
 - Key word here is “realisable” or “Imaginable” by the learner
 - In Dutch, “to imagine” = “zich REALISEren”
 - Context Problem (help pupils develop mathematically)
 - Mathematical concepts are developed by mathematisation process: [Conceptual Mathematisation](#)
 - [Models](#): bridge the gap between the informal and the formal
 - [Guided reinvention](#)
- Learner is a human being with potential for learning
- Learners must be actively involve in inquiry and developing/constructing knowledge.
- Teacher is merely a facilitator and motivator

Nature and Characteristics of RME

Mathematical concepts are developed by mathematisation process that is starting from **context-link solution** then students gradually **develop tools** for mathematical understanding to formal level. The process of developing **mathematical ideas** and **concepts** starting from the real world is known as

[Conceptual Mathematisation](#)



Nature and Characteristics of RME

Models: bridge the gap between the informal and the formal emerge in students' **mathematical activities** might prompt interactivities that lead to a higher level of mathematical thinking.

The students will be **offered various models** or **ways of representing and working** with the problems.

The informal level, these model could take the form of **sketches, pictures, diagrams or long ways of recording a calculation.**

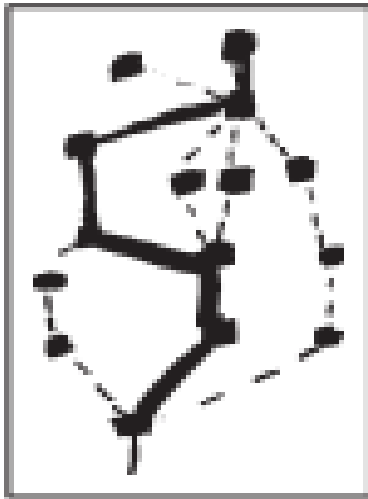
Five Tenets of RME

- The use of **context problems**
- The use of **models** or bridging by vertical instruments
- The use of pupil's **own productions and construction** (contribution)
- The **interactive character** of the teaching process (interactivity)
- The **intertwining** of various learning strands

(de Lange, 1996)

Hypothetical Learning Trajectories

- Learning route
- To provide teachers with a mental educational map to help them guide pupils
- It clarifies how abilities are built up in connection with each other



Hypothetical Learning Trajectory (Simon, 1995)

Hypothetical Learning Trajectory

- The learning trajectory is hypothetical because we can never be sure what they will do or whether and how they will construct new interpretations, ideas and strategies until they are really working on the problem.
- The trajectory clarifies how abilities are built up in connection with each other.

The Iceberg Model

$$7+28$$

Formal notation

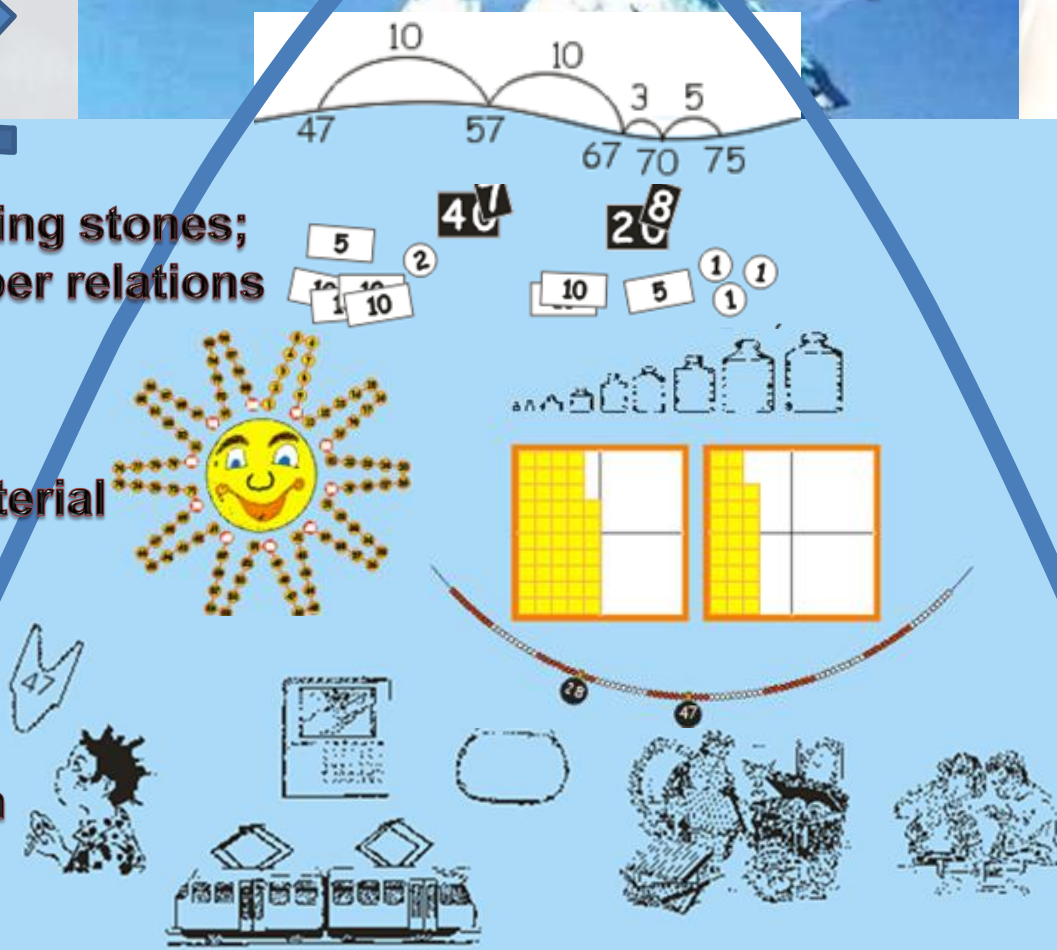
tip of the iceberg

building stones;
number relations

model material

mathematical
world orientation

floating
capacity



Iceberg Model

- To support teacher thinking about learning processes and strategies used by students.
- It is a visual model to distinguish the role of **informal, pre-formal, and formal representations** used by students.

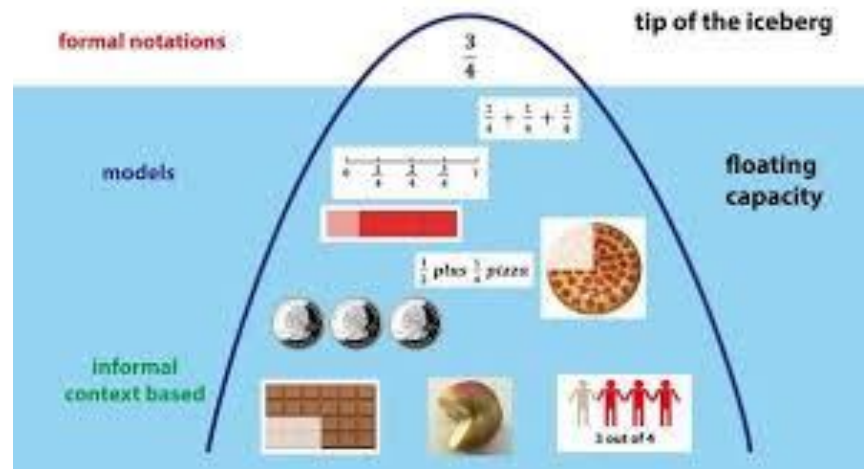
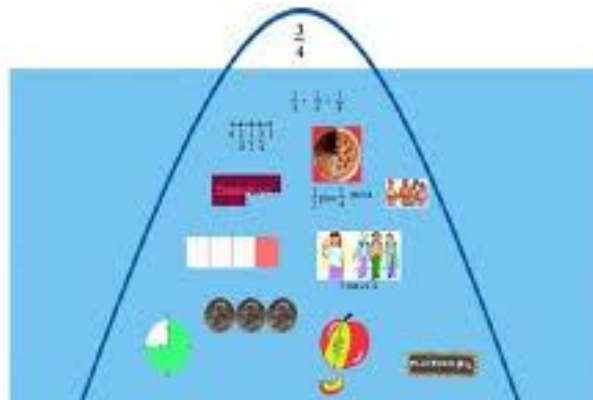
Iceberg Model

- Consists of the **tip of the iceberg** and a much larger area underneath, **the floating capacity**.
- Tip of the iceberg represents **formal mathematical goals**.
- The **pre-formal** representations are at the water line and informal representations are at the bottom.

Iceberg Model

- The **floating capacity** suggests that underneath the formal mathematics exists support student understanding of mathematics.

RME - The iceberg model



Use of characteristic materials

- Flash cards
- Numeric cards
- Number snake
- Numbers in love
- Sun game
- Etcetera...



Realistic Mathematics Education Emphasis on understanding and problem solving using real-life contexts

A box of oranges

Big Ideas

- Commutative and associative properties of multiplication
- Relationship between surface area and volume
Determine the relationship between the height, the area of the base, and the volume of a rectangular prism, and generalize to develop the formula (i.e., Volume = area of base x height)



Strategies

- Using repeated addition
- Skip-counting
- Using ten-times
- Doubling and halving
- Factoring and grouping flexibly

Model

* Open array

Three Heuristic Principles of RME

Three heuristics principles

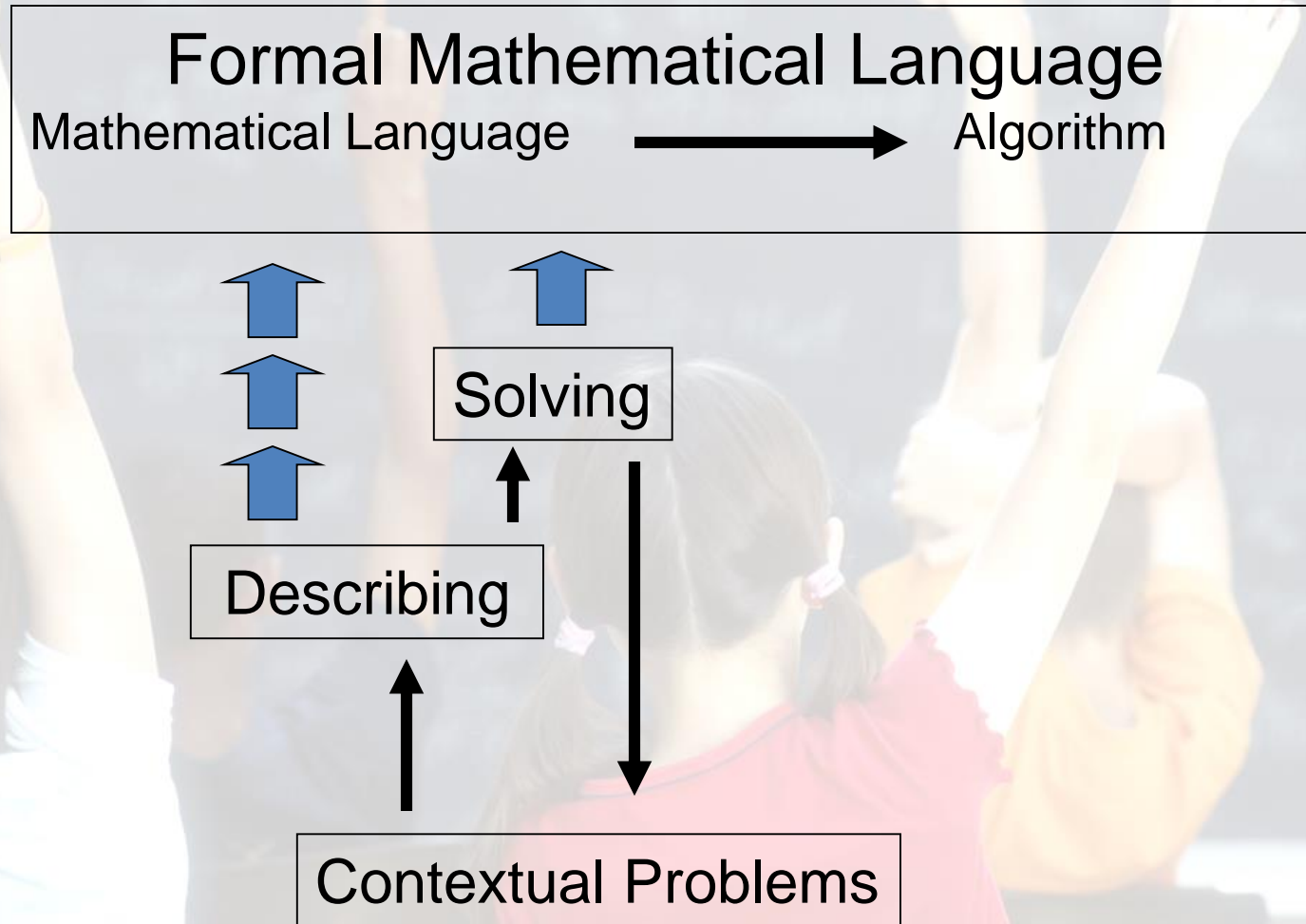
- **Guided Reinvention** through progressive mathematisation
- **Didactical Phenomenology**
returns us to the world from which we have abstracted.
- **Self-Developed or Emergent Models**
models as Tools for Thoughts

Three Heuristic Principles of RME

Three heuristics principles

- **Guided Reinvention** through progressive mathematisation
 - Well-chosen **contextual problems** so that offer students opportunities to develop informal, highly context-specific solution strategies.

Guided Reinvention



(Gravenmeijer, 1994)

Three Heuristic Principles of RME

Three heuristics principles

- **Didactical Phenomenology:** start from phenomena meaningful to the students

(returns us to the world from which we have abstracted.) should fulfill 4 functions:

- *Concept formulation*
- *Model formulation*
- *Applicability*
- *Practice*

Three Heuristic Principles of RME

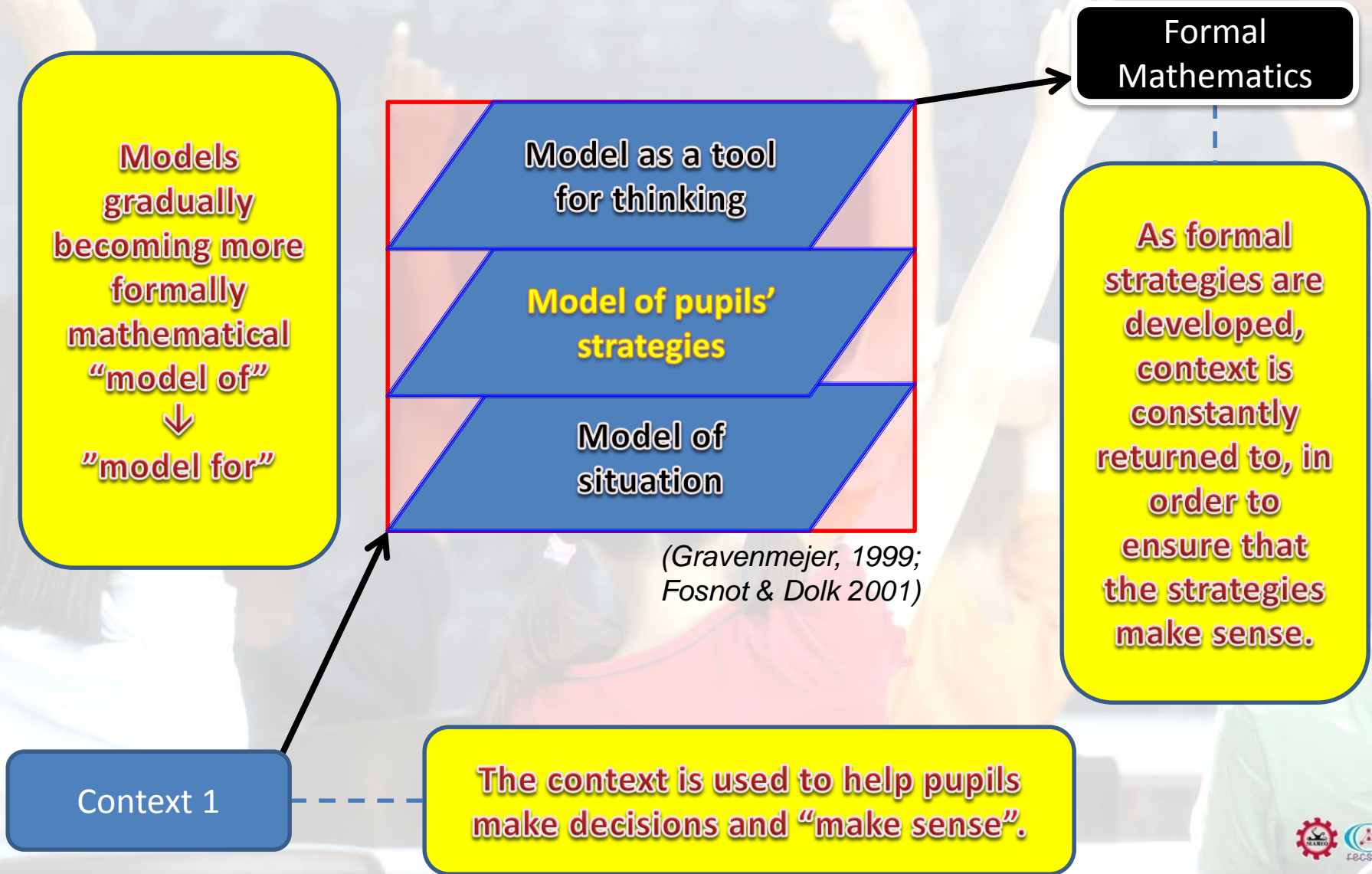
Three heuristics principles

- **Self-Developed** or **Emergent Models**

(models as Tools for Thoughts) bridging the gap between informal and formal knowledge.

In order to achieve this, the students need to be given opportunities to use and develop their own models when solving problems.

The Use of Models



Assessment

- Consider to be an integral component of the teaching and learning process.
- Doing assessment during the lesson such as asking the students to write an essay, perform an experiment, collecting data and to design exercises that can be used in a test, or to design a test for other pupils in the classroom.

Five Principles of Assessment

- Improve learning and teaching: measure the students during the teaching-learning process.
- Methods of assessment should enable the students to demonstrate what they know rather than what they do not know.
- Operationalise all the goals of mathematics education, lower, middle, and higher order thinking.

Five Principles of Assessment

- Quality of mathematics assessment is not determined by its accessibility to objective scoring. The students should be provided with tests that we really can see whether they understand the problems.
- Assessment tools should be practical, available to the applications in school cultures and accessible to outside resources.

Important Component

- Context independent testing
- Not only reflects the students' real world but also the real world of mathematics itself.
- Involve students' abilities to think and communicate mathematically.

Important Component

- Demonstrate higher order thinking skills associated with the discipline of mathematics.
- Offering situations in which allowed to work in their own ways to show creativity in their solution, portray a problem-solving attitude, and reveal their full learning potential.