ENHANCING MEANINGFUL TEACHING AND LEARNING OF MATHEMATICS THROUGH THE USE OF GRAPHICS CALCULATOR

Jerome A. Chavez
SEAMEO Regional Centre for Mathematics and Science
Malaysia

Allan Leslie White
University of Western Sydney
Australia

Cheah Ui Hock
SEAMEO Regional Centre for Mathematics and Science
Malaysia

Technology pervades current life and has influenced our educational institutions including the manner of instruction and the design of curricula. Such change needs to be evaluated in terms of the impact upon both teaching and learning. For example, the low cost, highly portable graphics calculator has become a feature of secondary school classrooms, yet are they being used mainly by teachers? How does the graphics calculator affect the student's comprehension and understanding of concepts? Is the use of graphics calculators more effective compared to the traditional chalk-and-talk methods?

This paper discusses how the use of the graphics calculator changes the teaching and learning of quadratic functions in the secondary classroom. Firstly, the paper compares teaching with the graphics calculator to the traditional chalk-and-talk method. The second part presents and analyzes the perception of students to the graphics calculator in their learning of mathematical concepts. It further discusses the findings based on data including students' interviews that reveals how the use of graphics calculator enhanced their learning. Finally, the paper advances several recommendations for pedagogical strategies when using graphics calculators.

Introduction

The primary purpose for teaching mathematics is to enable students to learn and appreciate mathematics in the best way possible. With creativity, passion, and resources available to them, teachers are able to implement various techniques and strategies in the classroom to make learning more meaningful and interesting to their students. In many classrooms, the usual way of teaching is the chalk-and-talk method. Teachers give the input verbally or write on the board and the students follow their instructions. However with the entry of technology into the classroom, the teaching of mathematics is changed (Simonson & Dick, 1997). Technology, specifically graphics calculator, has been widely adopted by academic institutions and has influenced the pedagogy in the classroom. For example, while graphics calculators were designed as personal tools, research by Cavanagh (2005) reported that students tended to use them as a shared device. He found graphing calculators played an important role in group activities as a kind of conversation piece for sharing mathematical ideas and making thought processes publicly available in the classroom. The technology facilitated social interaction in the classroom because it acted as a common point of reference for students as they discussed their ideas and results. Other researchers such as White (2004) have claimed that the graphics calculator has the potential to be a pedagogical Trojan Horse, subtly influencing a change in the usual teaching practices.

The concept of a function underpins many further topics in secondary school and higher level mathematics such as the study of calculus. "In Malaysian contexts the concept of function is introduced early at secondary level in Form 3 ... These ideas are introduce (sic) again in Form Four in Additional mathematics subjects" (Abdullah & Saleh, 2005, P. 254). The graphing of functions is an essential component of the study of quadratic functions. The graphs show different characteristics and properties that are not evident in an algebraic expression. However, students find sketching graphs of functions difficult and confusing. Abdullah and Saleh (2005) studied eight high ability Malaysian students and reported that their understanding of functions was flawed. They recommended that ways should be found to “make learning of mathematics especially those related to functions more accessible and meaningful if they are to be useful in higher learning” (p. 262). Are graphics calculators the way because they can potentially present concepts as more challenging and visually fascinating objects of knowledge? An examination of the research reveals there is a lack of findings concerning the teaching of quadratics using graphics calculators.

What research does reveal is that the usual classroom approach can be time-consuming and tedious with students’ focusing upon plotting individual points rather than concentrating upon the characteristics of the graph, whereas the graphics calculator gives fast graphs using either points or algebraic equations. "Previously, by the time students had drawn a graph, the class was nearly over - now they can talk about what it means" (Yang, Butler, Cnop, Isoda, Lee, Stacey, Wong, 2003, p. 60). Of issue in this paper are the reactions of students as they strive to develop an understanding of the general properties of the different functions while using a graphics calculator. Do the students report the graphics calculators enhance their construction of meaning in the teaching and learning process?

On the negative side of this question is the evidence of a lack of uniformity of teaching strategies. Mitchelmore and Cavanagh (2000) examined student understanding of function graphs when using graphics calculators. The sample consisted of 25 Australian Years 10 and 11 students from five schools who had used graphic calculators for 6-12 months. The results indicated that students had a weak understanding of the effects of scale-changes on a graph, of numerical accuracy, and accepted graphs as they saw them. The researchers concluded that misinterpretation of calculator graphs and the effects of scale on graphs needed to be explicitly addressed if students were to develop appropriate ideas and strategies for working with the technology. They also reported a tendency of students to accept the graphic image uncritically, without attempting to relate it to other symbolic or numerical information. While the sample is small, it is necessary to consider the issues raised and pay attention to the pedagogical practices.

Two Teaching Strategies Compared

The quadratic function is an important topic in mathematics. The graphing of quadratics is developed via stages that gradually increase the complexity of the function. Table 1 lists a possible sequence of the stages that teachers in a typical secondary school classroom follow to teach and graph a quadratic function.

A teacher using the chalkboard method will use a process illustrated in Figure 1, a diagrammatic representation of a non-calculator chalk-and talk pedagogical strategy. The leftmost column shows the algorithm of sketching the graph of a quadratic function and the stages in teaching quadratic function are described. In the middle column, the observable skills of the students are described. The right column shows the expected outcomes of the tasks.

Table 1: *An investigation of the properties of the graphs of quadratic functions can be done as shown.*

Forms of the Quadratic Functions	Properties to be investigated
$y = ax^2$ $y = -ax^2$	if the value of *a* changes
$y = x^2 + c$ $y = -x^2 + c$	if the value of *c* changes
$y = (x + c)^2$ $y = -(x + c)^2$	if the value of *c* changes
$y = (x + c)^2 + k$ $y = -(x + c)^2 + k$	if *c* and *k* change

A teacher using the chalkboard method will use a process illustrated in Table 2, a non-calculator chalk-and talk pedagogical strategy. The leftmost column shows the algorithm of sketching the graph of a quadratic function and the stages in teaching quadratic function are described. In the middle column, the observable skills of the students are described. The right column shows the expected outcomes of the tasks.

Table 2: *Chalk & Talk Method of Teaching Graphing of Quadratic Functions*.

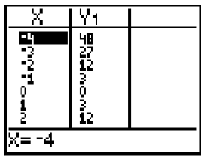
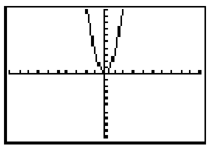
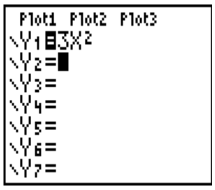
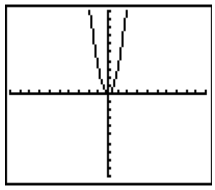
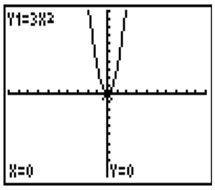
Teacher's Activities	Student's Activities	Outcomes
Write equations in the form $f(x) = ax^2$.	Make descriptions of the quadratic equations given.	$f(x) = 3x^2$
Generate values for x and y.	Compute coordinates of the points repeatedly.	
Plot the points on a coordinate axis. Sketch the graph.	Locate the points on the coordinate plane and sketch the graph. Identify vertex, direction of the opening of the curve, length of opening, axis of symmetry.	

Table 3 illustrates teaching with the use of a graphics calculator. The left column describes the steps a student needs to perform using a graphics calculator. The right column shows the expected outcomes displayed on the screen of a graphics calculator.

Table 4 lists the equivalent processes needed to teach a concept or illustrate a step towards graphing a quadratic function. From the table, it can be seen that both methods follow certain basic procedures, which procedures are crucial to the development of the graphs. Learning-wise, the students are expected to experience the same things regardless of the teaching method used. The students are required to record or observe the sketches of the graphs, make observations on the features and characteristics of the graphs, and make inferences about the shapes of the graphs, the intercepts and the various transformations of the graphs as the values of parameters change.

Table 3: *Using a Graphics Calculator to Teach Graphing of Quadratic Functions*.

Activities	Outcomes
Enter equation in ο editor.	
Press σ to display sketch.	
Press ρ to identify coordinates of vertex and other points.	

It is clear from Table 2 that the non-calculator method requires more time from students to do the actual sketching of the graph. Aside from having to take more steps (at least three versus just two using the calculator method), the students have to do the basic arithmetic to calculate values based on the function. To observe graphical transformations of the quadratic function, students using graphics calculator only need to perform a single step (entering a new equation or a new value, or pressing a button). Those not using calculators, on the other hand, will have to perform at least three steps for each of the transformations they want to observe.

Further, the chalkboard method can only approximate scales and curves. The graphs constructed may be a rough representation of the equation and often needs further smoothing of the curve. Care is needed in doing the scales so that correct coordinates of the points are identified. In contrast, after two steps with the use of the graphics calculator, the students are immediately presented with a smooth graph in the proper scales.

With the conventional mode of doing graphs, the students spent more time on computing for the values of the coordinates, plotting and identifying the points, smoothing the curves and scales. Students still struggle with computing the values of the coordinates which is essential in sketching a smooth graph. Any error in any of the coordinates will result to a bad graph. This situation is further complicated by the fact that the locations of points on the scale are simply approximates.

With the use of a graphics calculator, the scale of the points and units are accurate. These scales can be defined as desired by the student with simple manipulations on the graphics calculator. The sketch of the graph can be displayed instantaneously and through this sketch various properties of the graph can be investigated and explored.

Table 4: Comparison of the Steps Using the Chalkboard and Calculator Methods

Non-Calculator Method	Calculator Method
To make a sketch of a graph, the following procedure is followed: Calculate values for the variables. This identifies the coordinates of the points. Plot each point on the scale. Sketch the graph.	Enter equation in ο. Press σ. Sketch of the graph is displayed on the screen.
To observe the length of the opening of the curve, the same procedure as described above is repeated a number of times for different equations.	Enter different equations in ο. Sketch of the graphs will be displayed when σ is pressed.
To find the directions of the opening of the curve, sketches of a number of graphs with different values of *a* will have to be made. To do this the same process as above will be done.	Equations with different values of *a* can be entered with the other equations already in the ο editor. The graph is displayed once the σ is pressed.
To find the vertex, location of the coordinates can be traced through the graph.	Coordinates of the points can be located by pressing ρ.
To observe the behavior of the graph along the x-axis or y-axis, several sketches of the graphs have to be made and compared.	Sketches of different graphs can be displayed on the screen simultaneously. Observation of the characteristics and behavior of the graphs can be done easily.

Students' Reception to Learning Quadratic Functions Using the Graphics Calculator

This paper aims to consider the reactions of students to a lesson on quadratic functions using the graphics calculator, and to assess their receptiveness to learning the lesson, based on a lesson teaching done by one of the authors. The sample consisted of 31 Form 4 students from a secondary boys' school in Penang, Malaysia. A series of lessons on quadratic functions were given to the class following the sequence in Table 1.

The study consisted of a pretest, the actual lesson and a posttest. There were three sessions with the class. The first session was the administration of the pretest. Questions in the pretest were based on the topics the students found difficult as reported by the subject teacher. The pretest contained questions on inequalities, simultaneous equations and graphs of quadratic functions. The results revealed that students had more difficulty with sketching graphs of quadratic functions as compared with the results of the other two. The second session was the actual lesson teaching on sketching the graphs of quadratic functions. The actual teaching consisted of two parts: teaching the students how to use the graphics calculators and the lesson on graphs of quadratic functions using the graphics calculator. The second session lasted for two hours which included the conduct of the interviews with the students. The third session was the post-test. The posttest was given to students to identify whether there has been an improvement in their performances using the graphics calculator. The posttest contained questions on sketching the graphs, identifying the opening of the graphs, and finding the axis of symmetry. In the pretest, the students were not allowed to use the graphics calculators but they were allowed to use the graphics calculator during the posttest. After the teaching sessions and the posttest, the students had to complete a set of open-ended questions on how they felt about the lesson using the graphics calculator. These questions (See Table 3) were designed to assess the students' perceptions and assess these against scores in the pre- and post-tests.

Table 5. *After-Lesson Questions*

1.	What do you think about using a graphic calculator in learning mathematics? Does it help you to understand mathematics? In what way?
2.	How does the calculator help you to do mathematics?
3.	Do you want to use the graphics calculator in studying mathematics?

Aside from the open-ended questions administered, the students were likewise interviewed. Students' responses with the open-ended questions and the interviews were collected, grouped according to content and then labeled into categories. Students' responses were categorized broadly based on their perception of: (1) time, (2) visualization, (3) ease or facility, (4) understanding or achievement, (5) fun and social interaction and (6) issues. Whether these perceptions correspond to the students' actual receptiveness to the learning of the concept of quadratic functions is assessed later in the section and in the conclusion.

1. *Time*

   This category refers to students' responses that the use of graphics calculator saves time. Students learned fast and at their own pace. Students claimed that they do not need to wait for the teacher to explain the concepts; they could explore these concepts on their own. Moreover, the students did not have to wait for the other students who were slow.

   "Yes, graphic calculator help me to understand mathematics. It helps me *to save time* and it gives me some new ideas to explore mathematics."

   "By using graphic calculator, I think it will improve the result of the student in exam cause it more easy and no need to waste much time in drawing graph."

   "(*Ya,kerana boleh menjimatkan masa dalam membuat soalan.* ***)*** Yes, because it can save time in answering the question."

2. *Visualization*

   This refers to responses that the use of graphics calculator helps students to visualize concepts. The screen of the graphics calculator can show the graphs clearly and instantaneously. The students maintained that they could see exactly what happened with the graphs when they changed a value in the function.

   "It helps to learn more about drawing graph. It helps me a lot to understand mathematics."

   "Yes, it helps me a lot to this chapter. In the fun and harmonie class. The calculator give us to know how the graph is explain like shape going up or down."

   "Yes, it made the graph, to understand."

"(Ya, kerana boleh menjimatkan masa dalam membuat soalan). Yes, because it can save time in answering the question."

3. *Ease, Accuracy and Facility*
   This refers to students' responses that the use of graphics calculator yields more accurate results; facilitates calculation and plotting of graphs. The students maintained that they could try several times if there were mistakes. The students could immediately verify if their answers were right or wrong.

   "It helps us to save time. Less human errors."

   "(dapat menyelesaikan soalan dengan cepat dan tepat). It can solve the problem quickly and accurately."

   "It's make the questions is very easy and we cant get any wrong if use this graphic calculator."

   "Of course yes! Cause it can improve my maths skill. By using this calculator maybe I can solve any question of the maths with no problem."

   "The calculator help me to calculate more numbers of a question."

4. *Understanding and Achievement*
   This refers to students' responses that the use of graphic calculators aids them in studying and helps them to improve academic performance; makes them to explore more mathematics and helps them to understand the concept easily.

   "Of course yes! Cause it can improve my maths skill. By using this calculator maybe I can solve any question of the maths with no problem."

   "(Kalkulator ini dapat membantu saya menjawab banyak soalan matematik). This calculator can help to answer many questions in mathematics."

   "Yes, I will. The graphic calculator can help me much in education and many more because of it small, easy, comfortable and affordable (cheap)."

   "Yes, graphic calculator help me to understand mathematics. It helps me to save time and it gives me some new ideas to explore mathematics."

   "The graphic calculator help me a lot! It brings me more education in math. The graphic calculator cant make me boring because of the function."

   "It so fun to use a graphic calculator. Yes, it helps me to understand mathematics. It helps me the effect of the value a, band c of the graph."

"It helps to draw graph with a true vertex and get a real coordinates and know more about maths."

5. *Fun and Social Interaction*
   This refers to students' responses that the use of graphics calculator is fun and interesting. The use of graphics calculator becomes a reference where students can exchange and discuss ideas.

   "Yes! Because it is easy to use and make mathematics fun."

   "The graphic calculator help me a lot! It brings me more education in math. The graphic calculator cant make me boring because of the function."

   "Yes. It very help me to study. It is very interesting to use it. I would glad if this calculator can be used in our school."

6. Issues
   This refers to students' responses that the use of graphics calculator offers some disadvantages to students. The concerns shared by the students were valid and related to accessibility and affordability.

   "It is not affordable, expensive."

   "What if I do not have access to a graphic calculator?"

   "The teacher does not allow the use of graphic calculator in class."

   "It makes learning more difficult-too many steps to follow."

Juxtaposed against pre- and posttest scores, the qualitative responses of students indeed validate some of their perceptions.

Results of the pretest showed that students have difficulty with sketching the graphs of quadratic functions. During the pretest, the mean score for graphing quadratic functions was 0.52 and an SD of 1.29. In the pretest, almost all of the students were not able to sketch the graph, identify the value of a, identify the direction of the opening of the curve and find the axis of symmetry. However, after the intervention there was an improvement in the performance of the students. All of them were able to sketch the graph and identify the value of a and identify the direction of the opening of the curve. Most of them were able to find the axis of symmetry. Results of the posttest showed a remarkable improvement in the performance of the students in sketching the graphs of quadratic functions using the graphics calculator. The mean score of the posttest is 8.10 and an SD of 2.79. The total marks of the tests were 16.

Because the students used graphics calculators in the posttest, readers should not interpret the higher mean score in the post-test, as compared to that of the pretest, as an indication of an improvement in students' understanding of concepts of quadratic functions in any way.

The differences in scores indicate that basic arithmetic can still be a stumbling block towards inductive math problem solving. Students spent too much time computing the coordinates of the points with the given functions. Common errors were on computations and signs. As with graphing, students have difficulty identifying direction of the opening of the curve just by observing the value of *a*, and the axis of symmetry. As mentioned above, students were not successful in sketching the graphs of the functions because identifying the direction of the curve and the axis of symmetry is essential in doing so. The use of the graphics calculator makes it possible for students to skip this possible obstacle because the basic computations are already done for them. Visualizing the graphs enables the students to immediately create a mental picture for the same form of the function.

In a study such as this, findings cannot be generalized to a wider population. At best they provide initial insights into the way students experience the graphics calculator in their classroom. The graphics calculator definitely catches the attention and increases the receptiveness of students to a complicated lesson. It also cuts the time for calculation and arriving at values, giving both teacher and students to go into the more complex topics of graph behavior and maybe even the practical implications and uses of such graphs. The pre- and posttest scores show significant results when it comes to the more complex topics, although posttest scores were higher. This means that while the calculator aids in the actual computation and representation of graphs, deeper understanding requires more. Still, the students need to verify whether the graph being shown in the screen of the graphics calculator is the appropriate graph. The particular tests administered for this set of sample is limited in that it aims to measure differences in the students' performance when teachers use graphics calculators in teaching and/or students are allowed to use them in tests.

Ways Forward for Teaching Math

At the very least, the use of the graphics calculator in the classroom represents a novelty for young students who are exposed to technologically advanced gadgets at home and at play. If only for this, students' interest is aroused. Juxtapose that with the pace at and the ease with which the graphics calculator can produce a representation (graph) of a quadratic function, and students start paying serious attention. The teaching of mathematics requires time and concentration, and getting the students' attention is a significant first step towards achieving that.

Beyond that, the significant results in pre- and posttest scores and the items for which they are significant show that graphics calculator facilitates achievement in mathematics. Thus the results show that having access to the graphics calculators improves performance upon tests. The use of graphics calculator frees the students from the conventional routine of doing graphs and shifts learning from the usual teacher-centred approach to active student-centred learning approach. Students are given opportunity to be constructors of their own learning. Experiencing knowledge is one way of enhancing meaningful learning. Moreover, students exhibit more confidence and positive attitude towards learning mathematics. The issue of whether the students' develop a deeper understanding requires further study.

There are issues such as accessibility and affordability that may be considered. Students may be provided with the graphics calculator in school but they need access of these calculators outside of school. Students need to explore the features of the graphics calculator and how these features can influence their learning of mathematics. Free play and investigation can lead to deeper understanding. The students need to familiarize themselves with the various features of the graphics calculator for optimal benefits. Without access to graphics calculators during their private study time implies a higher expectation from students – that understanding which they seek to fast-track (by skipping basic computation to arrive at values, for instance) with the use of the graphics calculator will still be translated into approximate performance when the aid is not available.

Those who fear that the graphics calculator will make students lazy need to remember that they bare merely a tool developed and used to aid teachers and students alike in fostering understanding of mathematics. Graphics calculators do what they are programmed to do. Students still need to have a strong background of the fundamentals of mathematics, whether they have access to a calculator or not. Understanding of mathematical concepts should be inherent in what the students visualize and perceive. Thinking is performed by the students and not by the graphics calculators.

Students can learn mathematics in any manner available to them. This study does not attempt to replace the conventional mode of teaching with the use of graphics calculator. This study impresses that there are different ways of enhancing meaningful teaching and learning of mathematics and the use of graphics calculators is one of these ways.

References

Abdullah, S. A., & Saleh, F. (2005). Secondary students' understanding of functions: A case study. In H. S. Dhindsa, I. J. Kyeleve, O. Chukwu, & J.S.H.Q. Perera (Eds.). *Future directions in science, mathematics and technical education* (pp. 254-263) (Proceedings of the Tenth International Conference). Brunei: University Brunei Darussalam.

Cavanagh, M. (2005). Working mathematically: The role of graphics calculators. In M. Coupland, J. Anderson, & T. Spencer (Eds.), *Making mathematics vital: Proceedings of the 20th biennial conference of the Australian Association of Mathematics Teachers* . (pp. 80-86). Adelaide: Australian Association of Mathematics Teachers.

Mitchelmore, M., & Cavanagh, M. (2000). Students' difficulties in operating a graphics calculator. *Mathematics Education Research Journal, 12*(3), 254-268.

Simonson, L., & Dick, T. (1997). Teachers' perceptions of the impact of graphing calculators in the mathematics classroom. *Journal of Computers in Mathematics and Science Teaching, 16*(2/3), 239-268.

White, A. L. (2004). Can graphics calculators change pedagogical practices in secondary mathematics classrooms?. In W-C Yang, S-C Chu, T de Awis, & K-C Ang (Eds.), Proceedings of 9th Asian Technology Conference in Mathematics (pp.153-160). Blacksburg, VA: ATCM Inc.

Yang, W-C., Butler, D., Cnop, I., Isoda, M., Lee, P. Y., Stacey, K., & Wong, K. Y. (2003). A global view of curriculum issues on mathematics with technology: A panel discussion. In W.-C. Yang, S.-C. Chu, T. de Alwis, M-G Lee (Eds.), *Proceedings of 8th Asian Technology Conference in Mathematics* (pp.57-71). Blacksburg, VA: ATCM Inc.