Geometric Physical Models in Teaching Factoring Polynomials

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Abstract

Mathematics teaching is a never-ending struggle to help the students become mathematically literate, and develop favorable attitudes towards mathematics. This aim had hardly attained in the history of mathematics teaching. This study was conducted to determine the effectiveness of teaching 'factoring of polynomials' through geometric physical models to third year high school student at San Joaquin National High School (SJNHS). True experimental method using the pretest-posttest control group research design was used in this study. The IQ test, pretest-posttest teacher made test and survey questionnaire were administered to high school mathematics teacher. Validation of pretest-posttest was constructed based on the table of specification prepared by the researcher utilizing the Bloom's Taxonomy of objectives and was subjected for face validity among mathematics teachers. The selection of the final number of items was based on the level of difficulty of an item which had some relation with the discriminatory power of an item. In eliciting the data, two sections of the third-year students from SJNHS were utilized as the subject of the study, the experimental and control group both 40 student's respondents. As a result, both groups demonstrated improvement in their achievement in the four lessons on factoring polynomials. This showed that there was a significant difference between the means in the pre-test and post-test in lessons on factoring polynomials in both control and experimental groups. The study showed that the geometric physical models developed were suited to any level of intellectual ability among third year students of SJNHS. Further studies in the use of geometric physical models in the areas of Geometry and Algebra should be undertaken to determine what other areas can be taught effectively through the use of geometric physical model approach.

Keywords: *Geometry; Factoring polynomials; Physical models; Building blocks to support learning using mathematical manipulatives; Mathematics teaching exploring ideas in an active, hands-on approach; Honing mathematical thinking skills, connect ideas and integrate knowledge*

Introduction

Mathematics teaching is a never-ending struggle to help the students to become mathematically literate, to acquire mathematical concepts, skills and to develop favourable attitudes towards mathematics. It is for this reason that all educators have to exert efforts towards the improvement for mathematics teaching.

Background and Overview

For instance, over a thousand of local mathematics teachers and twenty foreign mathematicians converged at the Legend Hotel, Puerto Princesa Palawan, for the 12th Biennial International Conference on Mathematics Education organized by MATHTED on October 24-26, 2019 to search for solution to current issues and challenges or problems in mathematics education which might have caused the decline on mathematical literacy among our youth. It should be noted straightaway that many of today's changes and challenges manifest itself in many aspects, consequently, two of these changes deserve special mention: rapid technological advancement and fundamental social change

Apparently, the success of mathematics teaching depends upon the methodology used because "method of teaching is the lifeblood of mathematics". It is the most important role of a teacher in a classroom setting to generate interest and make the students learn the subject effectively. This will therefore require teachers to have a working knowledge of different methods which will make them more resourceful in meeting the situation arising in the classroom. Teaching mathematics can only be described as truly effective when it positively impacts student learning. We know that teaching practices can make a major difference to student outcomes, as well as what makes a difference in the classroom.

Review of Related Literature and Problem Statement

According to Young (1984) a mathematics educator and author of many books on mathematics education. "Equation is the backbone of algebra." In our own words, this means that the fundamental aim of teaching and learning algebra is to be able to solve equations. Unfortunately, this aim had hardly ever attained in the history of mathematics teaching. As revealed during the presentation by Reyes-Cruz (2014), the second ranking difficulty encountered by most students in mathematics is solving linear and quadratic equations. This very significant result is worth considering.

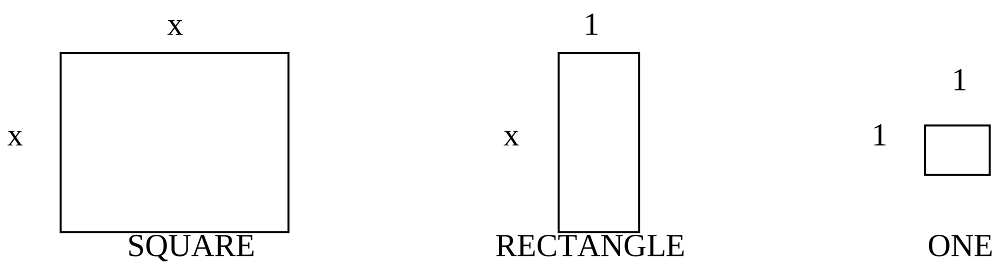
There could be many factors affecting this problem. One possible factor could be in the teaching method used. In trying to analyze this finding, one could easily be led to infer that possibly, the students were taught solving linear and quadratic equations without thoroughly understanding the polynomials and the principles involved in manipulating them. We could not, however, blame the teachers for that misfortune, for truly, this portion of the Algebra course has frequently been dull and boring. Algebra is thinking logically about numbers rather than computing with numbers. In algebra you are a second step of abstraction removed from the everyday world: those x's and y's usually denote numbers in general, not particular numbers. In algebra you use analytic, qualitative reasoning about numbers, whereas in arithmetic you use numerical, quantitative reasoning with numbers.

This section is devoted to give the reader information about the development of the physical models and the bearings of these models in this endeavour. These models are neither discoveries nor invention but rather, they are products of imaginative minds of persons whom the reader will come across in the succeeding paragraphs.

Similarly, models as physical objects (e.g plaster model of geometrical solids or surfaces) mental instantiations (e. g of axiomatic geometry systems) illustrate usages that lie outside our particular field of activity. Dimensional analysis (for example, Giordano @ Weir 1991) is an extremely powerful and neglected technique in modelling applications. It provides a way to introduce quite difficult physical models to those without a strong background in mathematics.

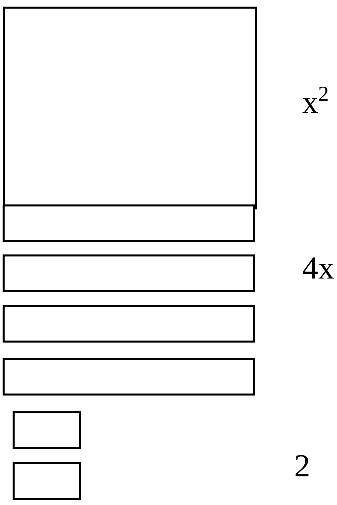
Originally, there was only one model. This was used by Z. P Dienes (1963) to teach factoring quadratic trinomials to a small group of eight-year-old children in a study described by Bruner (1968). This model was made of wooden blocks and is named "Dienes' Blocks" (1963) after its originator. Having this material, Bidwell modified the model by using strips of tag board (Figure 1) instead of subject matter in the elementary as well as in the high school. According to his own account, the method had a favourable effect on the students and was highly appreciated by teachers (Bidwell, 1972).

Figure 1 Strips of tag board

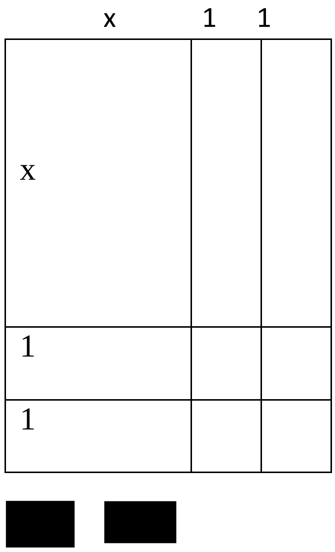


Again, in another issue of the "Mathematics Teacher" Gibb wrote an article about a further extension of Bidwell's strips. He found out that the same model can be extended to the case where the quadratic is not factorable over the integers but is factorable over the reals. His example shows how the students can lead to factor out a polynomial such as $x^2 + 4x + 2$. (Gibb, 1974). The students should be able to assemble the strips as shown in the following diagrams (*Figure 2 and Figure 3).*

Figure 2 Factored form of $x^2 + 4x + 2$ using strips of cardboard



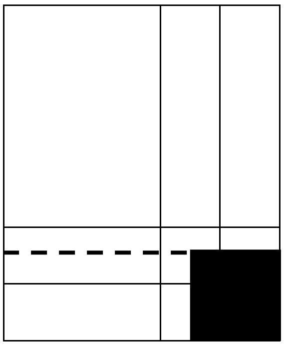
Factored form of $x^2 + 4x + 2$



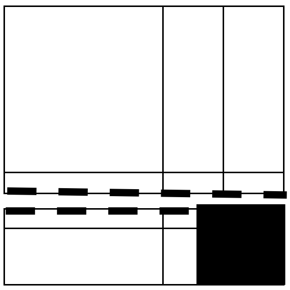
$x^2 + 4x + 4 - 2 =$

$(x + 2)^2 - 2$

Figure 3 Factored forms of the equation containing square roots



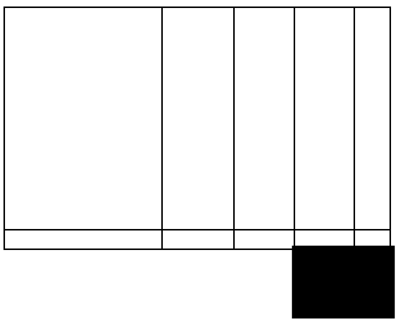
$(x + 2)^2 - (\sqrt{2})^2$



$x + 2 - \sqrt{2}$

$\sqrt{2}$

After factoring, equal to



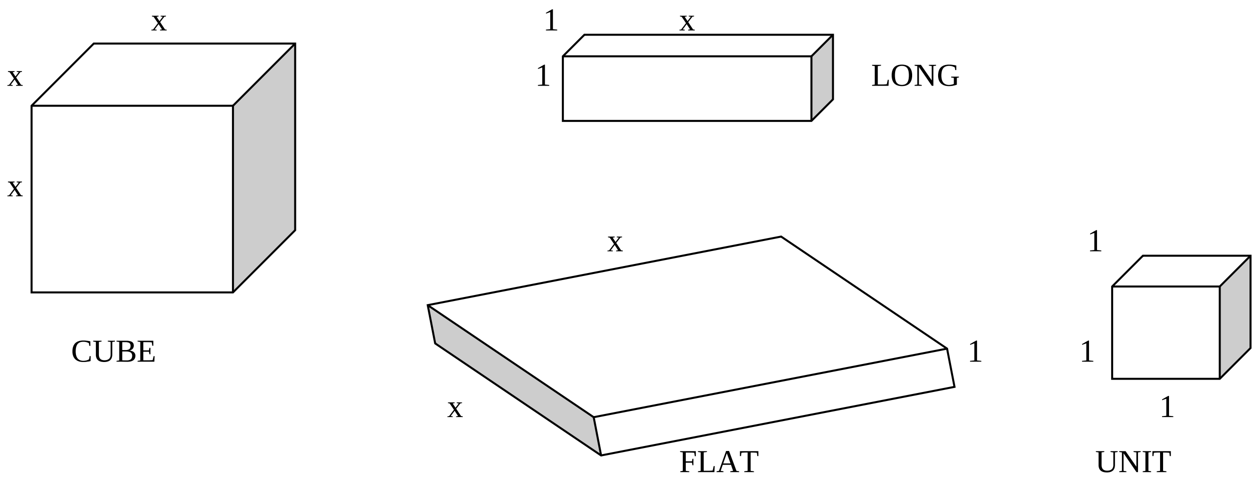
$(x + 2 + \sqrt{2})(x + 2 - \sqrt{2})$

The difficulties here, however, as Gibb (1974) himself pointed out are as follows:

1. If the student is restricted to the same pieces, it is obvious that no solution is possible.
2. The problem should be discussed after the introduction of real numbers.
3. The student must have been working with models for irrational numbers based on paper folding in order to construct, using scissors, the required black square of side √2 to get a rectangle of sides x *+ 2 + √2 and x + 2 - √2.*

The method therefore is not applicable to second year high school students. A worth considering extension of the Diene's (1963) blocks as suggested by Gibb is that of factoring cubic polynomials. *The model consists of 'x by x by x' cubes, 'x by x by 1' flats and 'x by 1 by 1' longs, and '1 by 1 by 1' units. (Figure 4).*

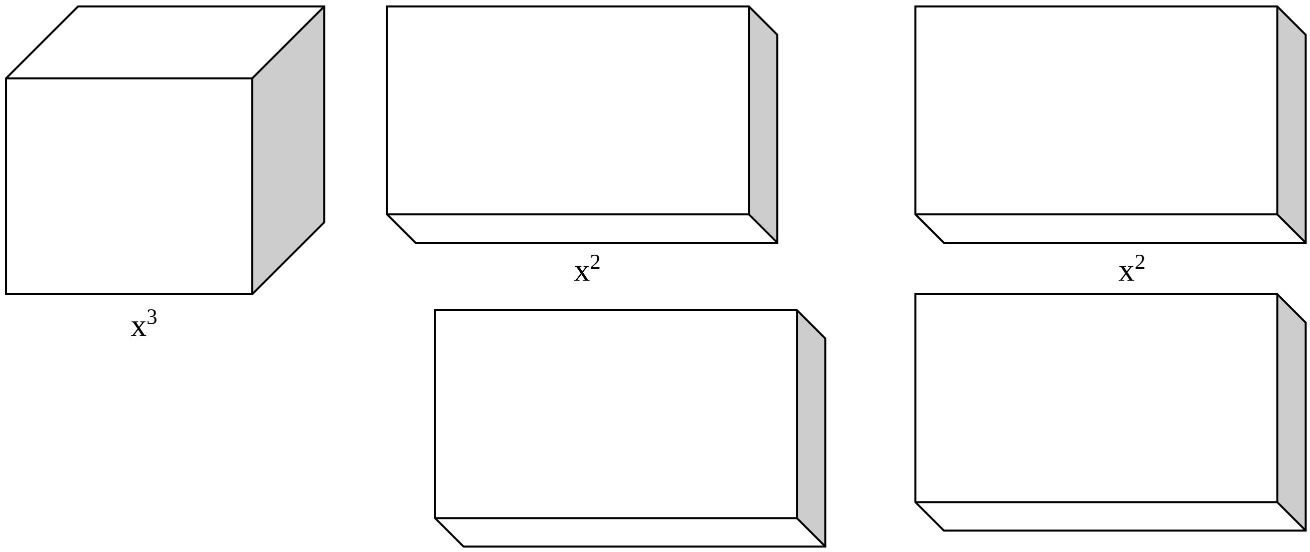
Figure 4 Kit containing rectangular blocks of wood



The following figures illustrate how these materials used to factor a cubic polynomial such as

$$x^3 + 4x^2 + 5x + 2.$$

Figure 5 Cubic illustration of the equation $x^3 + 4x^2+5x + 2$using rectangular blocks of wood



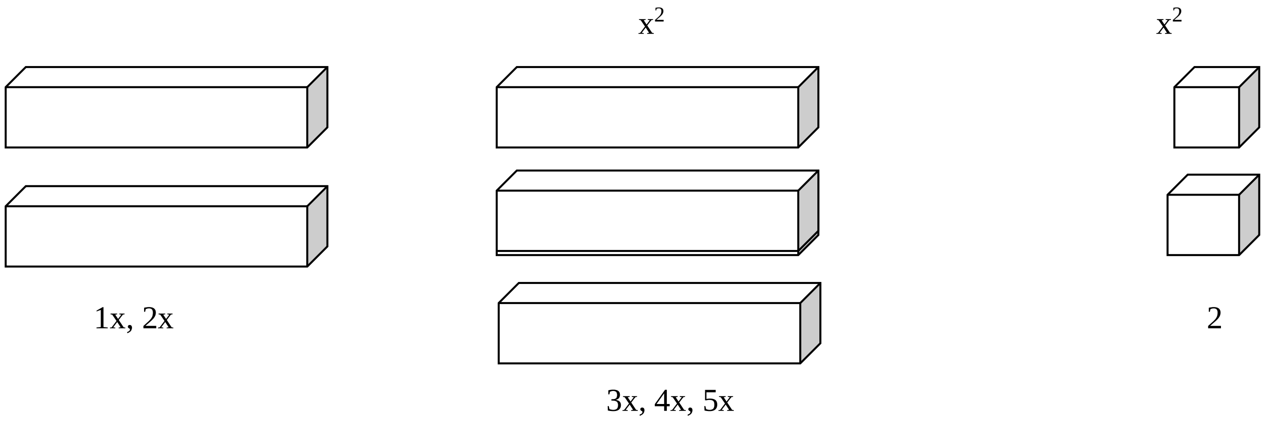
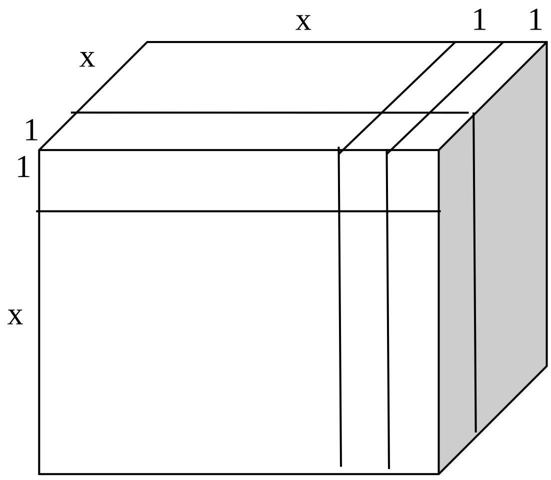


Figure 6 Model of the equation $x^3 + 4x^2 + 5x + 2$ using rectangular blocks of wood



(x + 1) (x +1) (x + 2)

Problem Statement and Rationale of Study

The reader should note that in the literature, the models were used for factoring polynomials only. No one of the aforementioned authors made any mention about using the models to multiply polynomials. They failed to consider the possibility and meaningfully by starting with multiplying polynomials.

Hence, this study aims to explore how physical models as illustrated in Figure 1 to Figure 6 can be used to teach factoring polynomials at secondary mathematics level with development of lesson exemplars and teaching modules.

The strips in Figure 4 are made of tag board with sides are of different colours (preferable black and white) in order to make possible the multiplication and factoring of polynomials which contain negative terms. The black side of the strips are used only when the polynomials under consideration contains negative terms.

For the convenience of the user, the ‘x by x’ strip is simply called a ‘SQUARE’; the ‘x by 1’ strip a ‘RECTANGLE’ and the ‘1 by 1’ strip as ‘ONE’. In the same manner, the ‘x by x by x’ cube is simply called a ‘CUBE’, the ‘1 by x by x’ block a ‘FLAT’, the ‘1 by 1 by x’ block a ‘LONG’, and the ‘1 by 1 by 1’ unit cube simply a ‘UNIT’.

Each of the models has its own function that is distinct from each other. Since the strips are two dimensions only (disregarding the thickness of the tag board), they are used for multiplying and factoring linear and quadratic polynomials, on the other hand, the blocks of wood having three dimensions are specifically used for multiplying and factoring cubic polynomials. The reader, who is interested to know these models works, is encouraged to go through the modular program.

However, the operational definitions of (a) monomial are polynomial which has only one term (b) trinomial it is an algebraic expression consisting of three terms (c) polynomial is an expression consisting of variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponentiation of variables

Research Objectives and Research Questions

The challenge that the schools have is not that we don’t know what teaching effectiveness is or that we do not have models and research to guide us. The challenge is how to ensure that these practices are in every classroom and in every teacher’s repertoire of professional practice. The foundation for a teaching effectiveness relies on the practice that uses the principles and behaviours of teaching. It is this alignment of practices in schools and where we need to improve if we are to reach the goal of having effective teaching in every classroom.

Thus, this study was conducted to determine the effectiveness of teaching factoring of polynomials through geometric physical models to third year high school student at San Joaquin National High School

Specifically, it sought to answer the following questions:

1. What is the profile of the third-year high school students in terms of their,
   1.1. Intellectual ability?
   1.2. Pre-test results in factoring of polynomials in the control group and experimental group by using:
      1.2.1 Common Monomial factors
      1.2.2 Perfect Square Trinomial
      1.2.3 Quadratic Trinomial, and
      1.2.4 Cubic Polynomials

2. Is there a significant difference between the means of the pre-test of the control group and the means of the pre-test of the experimental group?

3. What is the profile of the third year high school student in terms of their post-test results in factoring polynomials in the control group and experimental group in terms of

3.1 Common Monomial Factors
3.2 Perfect Square Trinomial
3.3 Cubic Polynomials?

4 Is there a significant difference between the means of the post-test of the control group and the means of the post-test of the experimental group?

5 Is there a significant difference between the means of the post-test of the control group with the means of the pre- test and post-test of the experimental group?

The following null hypothesis were tested in this study

1. There is no significant difference between the means of the post-test of the control group
2. There is no significant difference between the means of the pre-test of the control group and the means of the pre-test of the experimental group.
3. There is no significant difference between the means of the post-test of the experimental group

Methodology

Research Design and Instrumentation

To determine the effectives of using geometric physical models in teaching factoring polynomials, the true experimental method using the pretest-posttest control group research design was used in this study. Three sets of instruments were used in this study. They were the IQ test, administered by the graduate school psychometrician, pretest-posttest and the survey questionnaire to high school mathematics teachers within the locale of the study.

Validation and Piloting

Validation uses pretest-posttest questions that was constructed based on the table of specification prepared by the researcher utilizing the Blooms Taxonomy of objectives. It was subjected for face validity among mathematics teachers. The said test was tested among the 3rd year students of the other school. The result of the test was analyzed, some items were revised based on the item analysis. The selection for the final number of items was based on the level of difficulty of an item which had some relation with the discriminatory power of an item.

After securing a proper written permission from higher authorities, arrangements were made to the concerned teachers by the change of schedule of the students who were the subjects of the study.

Sampling, Data Collection and Analysis

In eliciting the data, officially enrolled students from the two sections of third year high school of San Joaquin National High School were the subject of the study. Random sampling technique was employed by the researcher to identify the forty (40) students' respondents for the experimental group, and forty (40) students' respondents for the control group. Identical pre-test on factoring polynomials were administered to both section before the geometric physical models were given to the experimental group and corresponding lessons to the control group. After going through the lesson, post-tests were administered to both groups. The pre-test scores and post-test scores of the two groups were compared using z-test for uncorrelated means while the difference of the pre-test and post-test were tested using the t-test for correlated means.

Statistical treatment of the data was employed in this study. Frequency and percentage distribution were employed in determining the number of the subjects of the study. The same were used to determine the IQ and gender/sex of the respondents. Mean and Standard Deviation were computed to determine the achievement of students in the pre-test and post-test of the two groups in terms of their mathematical skills. The Z-test two-sample were used for means at > .05 level of significance that was used to determine the significance of the mean difference between the pre-test result/post-test results of the experimental and control groups.

Findings and Discussions

The main focus of data collection in this research is to elicit responses based on the effectiveness of teaching factoring of polynomials through geometric physical models. The intellectual ability using the respondent general average found in their respective form 138 of the 41 out of 80 respondents or 51.25 percent of the third-year high school students in the experimental group and control group were below average, 20 out of 80 respondents or 25 percent was average and 19 out of 80 respondents or 23.75 percent was low.

Table 1 shows the means in the pre-test of the control group were 1.70, 2.62, 2.72 and 2.40 respectively, while the experimental group the means in the pre-test were 2.47, 3.47, 3.62 and 2.45 respectively. However, the means of the control group in the post-test were 3.55, 4.12, 6.45 and 5.08 respectively. In the experimental group, the means in the post-test were 4.20, 6.08, 8.25 and 6.12 respectively.

Table 1 Means of the Pre-test and Post-test Results for Both Control Experiment and Experimental Group

Lesson	Control Group		Experimental Group	
	Means		Means	
	Pre-test	Post-test	Pre-test	Post-test
1	1.70	3.55	2.47	4.20
2	2.62	4.12	3.47	6.08
3	2.72	6.45	3.62	8.25
4	2.40	5.08	2.45	6.12

As reflected in the post-test results, both groups showed improvement in their achievement in the four lessons on factoring polynomials. Table 2 shows the computed z-test of 4.71 of the pre-tests from the control group and the experimental group was greater than the tabular value of 1.96 at .05 level of significance with the mean difference of 2.75; hence, the null hypothesis was rejected. In the post-test of both the experimental group and control group, the 6.69 computed z-test was greater than the tabular value of 1.96 at .05 level of significance at 4.73 mean differences. Hence, the null hypothesis was rejected.

Table 2 The Computed Z-Test Result

	Control Group	Experimental Group
Computed Z-test	4.71	6.69

Tabular value at 1.96>.05 level of significance

Table 3 shows tn the control group, the computed t-test of 13.92 of the pre-test and post-test results was greater than the tabular value of 2.02 at .05 level of significance, while in the experimental group, the computed t-test or 21.53 of the pre-test and post-test result was greater than the tabular value of 2.02 at .05 level of significance, hence, the null hypothesis was rejected.

Table 3 The Computed T-Test Result

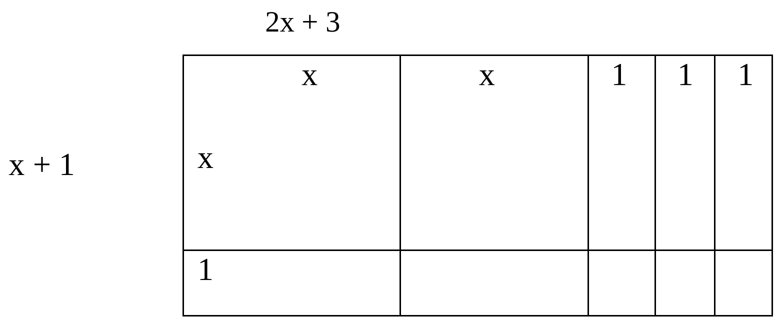
	Control Group	Experimental Group
Computed t-test	13.92	21.53

Tabular value at 2.02 > .05 level of significance

This showed that the null hypothesis was rejected; that there was no significant difference between the means of the pre-test with the post-test in the four lessons on factoring polynomials in both control and experimental groups. The acceptance means that there was no difference between the means of the pre-test to the post-test in the four lessons on factoring polynomials.

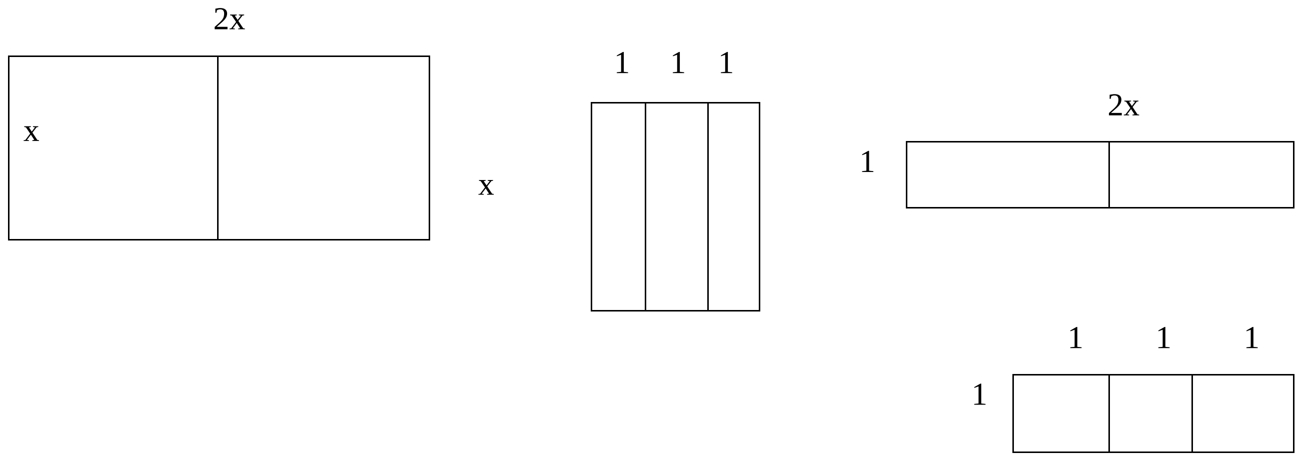
With the models, the students know, for example, that (2x + 3) (x + 1) = $2x^2$ + 5x + 3, but algebraically, he knows nothing about how the terms of the product can be obtained from the factors. The distributive law which is involved in multiplying and factoring polynomials can be taught by using the same materials. Let the students for the last time construct the rectangle representing the polynomials, (2x + 3) (x + 1) as shown in Figure 7.

Figure 7 Rectangle representing polynomials (2x + 3)(x + 1)



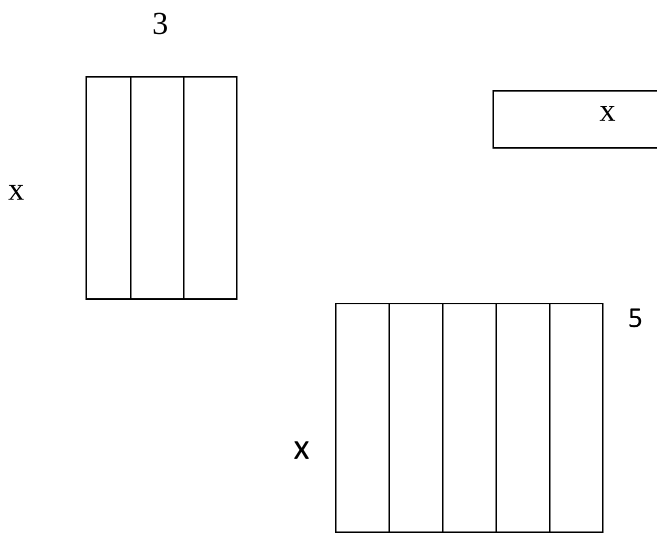
Then guide them to break this rectangle into four smaller rectangles which respective areas are $2x^2, 3x, 2x$ and 3. The students can easily see that the expression (2x + 3) (x + 1) and $2x^2 + 3x + 2x + 3$ are equal since they refer to the same rectangle. The teacher can now lead the students to discover what terms in the factors should be multiplied in order to get the terms $2x^2, 3x, 2x,$ and 3 as shown in Figure 8.

Figure 8 Four smaller rectangles in which the respective areas are $2x^2$, 2x, 3x and 3



To show that $2x^2 + 3x + 2x + 3$ equals $2x^2 + 5x + 3$ is not a problem since the rectangles can be combined (as shown in Figure 9) to form a bigger rectangle which the area is $5x$.

Figure 9 Combination of rectangles to form a bigger rectangle of which the area is 5x



The students can now be introduced to Distributive Property of Multiplication over Addition (DPMA)

(a + b) (c + d)

ac +ad + bc + bd

Conclusion

Implications and Limitations

This research project is significant because of its innovation in the teaching of polynomials through geometric physical models. Based on the findings of the study, and in spite of the fact that the intellectual ability of the third-year high school students of San Joaquin National High School was below average, they could still perform well in complex mathematical lesson like factoring polynomials through the use of geometric physical models if they were motivated. The use of geometric physical models enabled students to gain higher scores in the post-test hence, instruction using geometrical physical models was more effective than the traditional method of instruction in teaching factoring polynomials.

The study showed that the geometric physical models developed were suited to any level of intellectual ability among third year students of San Joaquin National High School and that they benefited in manipulating the models. The performance of the students exposed to geometric physical models of instruction differed significantly from the performance of the student taught by the traditional method in the post-test. Hence, geometric physical models of instruction were more effective than the traditional approach.

The reader must have easily seen that the model used in this module for multiplying and factoring polynomials have certain limitations. Not all polynomials can be multiplied or factored by using the same materials. For example, neither the strips nor the blocks can be used to find the product of polynomials such as $(x^3 + 1)$ and $(x + 1)$ which are the factors of $x^2 + x + 1$. Anyway, the students will not always be using concrete materials in mathematics and it is advisable that they should not. It should be remembered that the goal of mathematics is to use symbol and abstractions. Any physical model for any mathematical concept should therefore serve not as an end in itself but rather as a means to an end. The role played by the models in this study was simply to make the teaching learning situation meaningful, long-lasting, and enjoyable.

After completing the study, the teacher should guide and encourage the students to do away with the models. He should however be careful in guiding the students in moving from the concrete stage of manipulation to the purely symbolic stage. A sudden change of anything has always prove dangerous. In mathematics, it will only turn the students to confusion rather than the effect in learning. To achieve continuity in teaching multiplication and factorization of polynomials from this module to the purely algebraic operations, the following approach is recommended.

Recommendation

The researcher has tried it in a form of a demonstration lesson during the presentation of this paper. Only one thing can be said about the result. The reactions of the were exciting. This approach has been proven to have the advantage over the module and is, therefore, recommendable, the teacher has more control in leading the students to move from the purely physical stage of manipulation to a mixture of concrete and symbolic experiences finally to the purely symbolic stage.

Whichever method is used with the models, in the final stage of multiplying and factoring algebraically, then in solving problems which involve multiplying and factoring, the student will always have resorted to physical models which reinforce the area and dimensions of models for product and factors, respectively.

Based on the findings and conclusions, it is fitting that geometric physical models must be as a supplementary instructional material in teaching factoring polynomials. Further studies in the use of geometric physical models in the areas in Geometry and algebra should be undertaken to determine what other areas can be taught effectively through the use of geometric physical model approach. Geometric physical models' construction should be developed for teaching learning enhancement. Mathematics teachers should undertake training on geometric physical models' construction especially in factoring polynomials Administrators should provide financial support in the construction of geometric physical models in the class. Administrators should encourage all teachers to use instructional models such as geometric physical models in teaching especially in mathematics.

Significance and Contribution in Line with Philosophy of LSM Journal

This article contributes to the bulk of knowledge in geometric physical models for teaching and learning of factoring polynomials with exemplars through modular approach. Students are expected to engage in various robust cognitive and social learning tasks as well as using these tasks productively in mathematics learning.

Acknowledgement

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References

Bidwell, J. K. (1972). A Physical Model for Factoring Quadratic Polynomials. *Mathematics Teacher*, LXV, 3 201- 205.

Bruner, J. (1968). *Towards a Theory of Instruction*. New York: W.W Norton & Company.

Dienes, Z. P. (1963). *Base ten Blocks*, *Modern Algebra*. New York.

Gibb, A. A. (1974). More on Physical Models for Factoring Polynomials. *Mathematics Teacher*, LXVII, 133- 138.

Giordano, F.P., & Weir M.D. (1991). *Differential Equations*, *A Modelling Approach*. Menlo Park: Addison- Wesley. GSP, Geometers Sketchpad (n.d.)

Reyes-Cruz, F. (2014). *A Study of difficulties in Algebra among College Students at the Philippines Normal College 2012- 2013*. Unpublished Seminar Paper, University of the Philippines.

Young, J.W.A. (1984). *The Teaching of Mathematics in the Elementary and the Secondary School*. Longmans, Green and Company.

Appendix A

Lesson Exemplars through Modular Approach

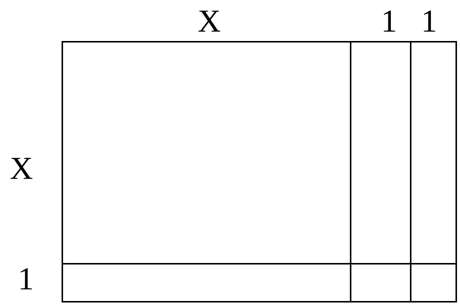
(I) Multiplication of Polynomials

In elementary mathematics, you have learned that the area of a rectangle is equal to the product of width and length and the volume of the rectangular building (cylinder) is equal to the width times the length times the height. For a square, the area is simply the square of the side and for a cube, the volume is simply the cube of the side. You have also learned that when two numbers are multiplied such as 3 times 6 equals 18, we call 3 and 6 as factors of the product 18.

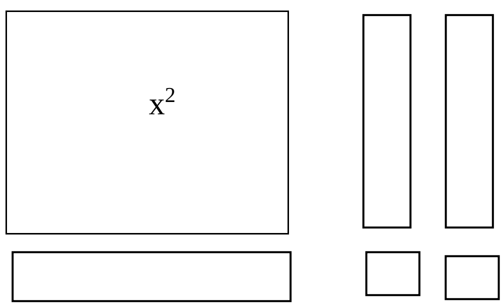
Previously we learned that linear polynomials with the edges or sides of the strips or of the blocks; quadratic polynomials with the areas of the strips or of the blocks; cubic polynomials with volume of the blocks. Oppositely, a given polynomials factors may be thought of as rectangles or rectangular blocks whose dimensions are the given factors.

Now, using the strips construct a rectangle whose dimensions are (x+1) by (x +2).

Answer



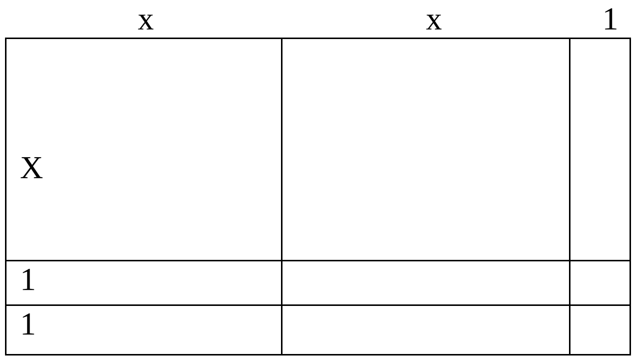
What is then the area of the rectangle you have constructed?



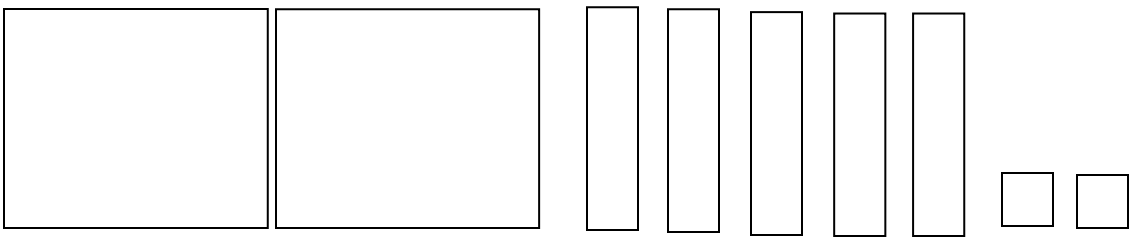
Answer: $x^2 + 3x + 2$

Construct a rectangle whose sides are given by polynomials **<u>2x+1 and x +2</u>** and then find the area

Answer:



Then the rectangle formed is consist of the following;



2 squares, 5 rectangles, 2 ones

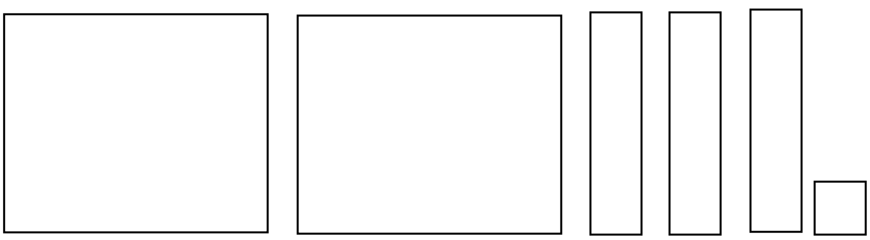
The respective areas of the strips used are $2x^2$, 5x, 2. Therefore, the area of the rectangle is $2x^2 + 5x + 2$.

(II) Factoring of Polynomials

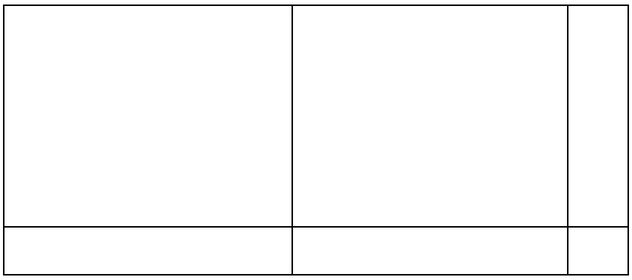
You are now about to begin factoring polynomials. The skills and techniques which you have garnered in the preceding phases will help you discover the process of factoring easier and simpler.

Start now by assembling the following materials to form a rectangle;

2 squares, 3 rectangles, and 1 one.



To form;



What is the area of the rectangle formed?

Answer: The area of the rectangle formed is equal to the sum of the areas of the component parts.

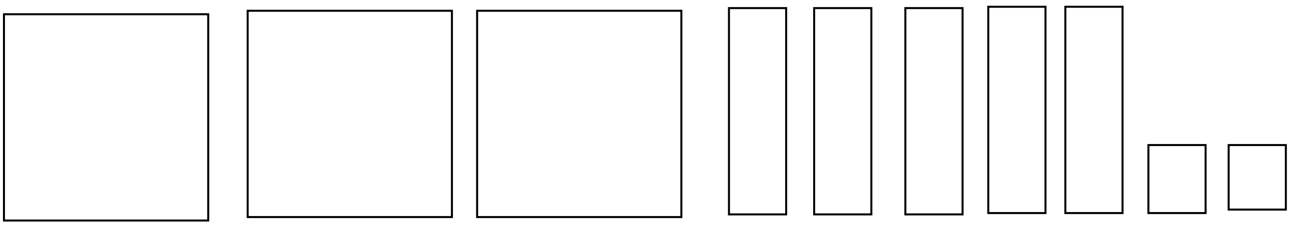
$2x^2 + 3x \ + 1$

What are the dimensions or lengths of the sides of the rectangles?

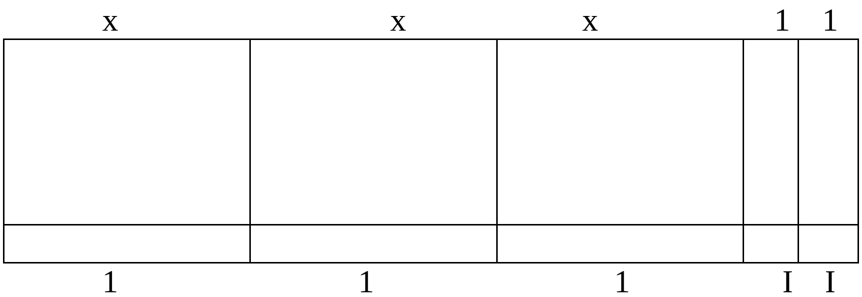
Answer: The dimensions of the rectangle are; Length (2x + 1) and the width (x + 1)

Another example: Construct a rectangle whose area is given by the polynomials $3x^2 + 5x + 2$.

Answer:



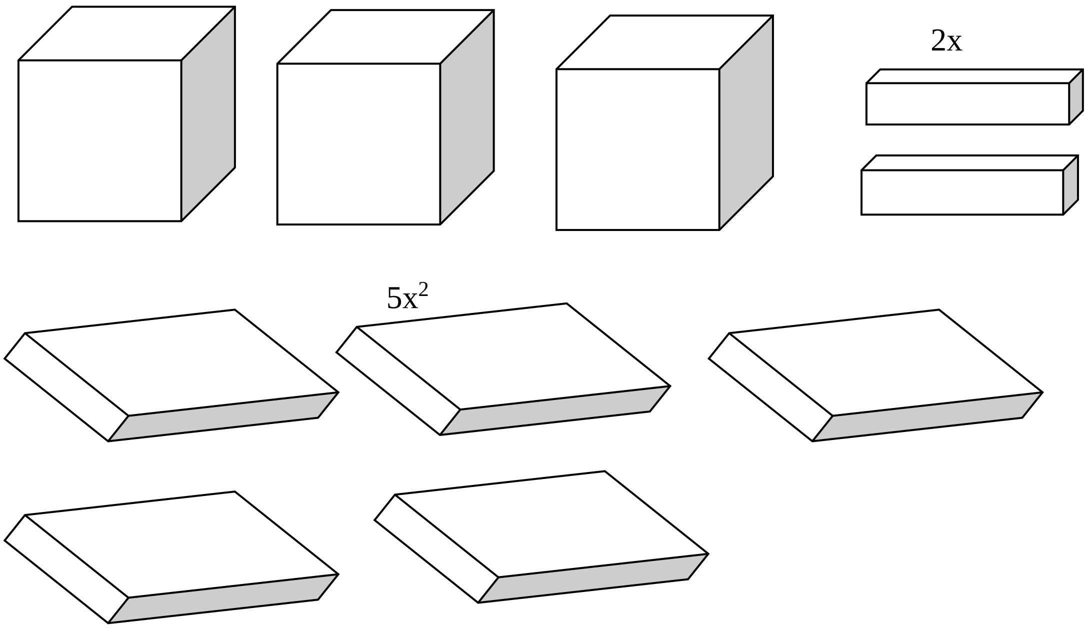
Then formed;



Find the dimensions of a rectangular building whose volume is $3x^3 + 5x^2 + 2x$.

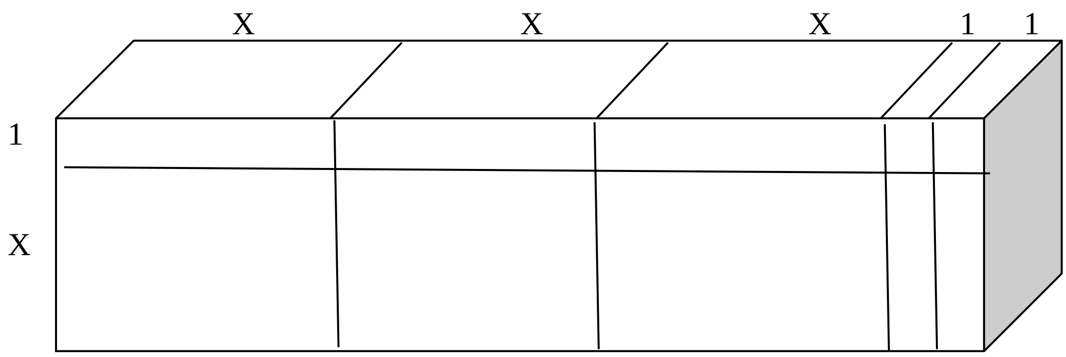
Answer:

$3x^3$



$$3x^3 + 5x^2 + 2x$$

Then formed



Dimensions: x by (x+1) by (3x +2)

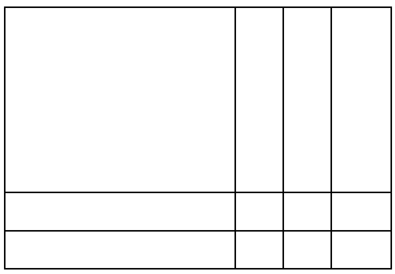
APPENDIX B

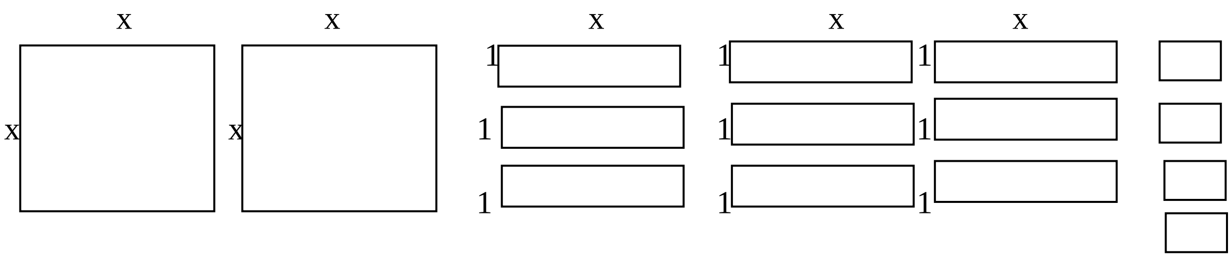
Sample Evaluative Pretest-Posttest

This test will measure how much you have learned from the module.

1. An edge of a box is a good and appropriate representation of a
   a. Linear polynomial c. quadratic polynomial
   b. Cubic polynomial d. both linear and a quadratic polynomial
2. An appropriate representation of a quadratic polynomial
   a. Ruler c. box
   b. Rectangular garden d. all of the above
3. Consider the following statements
   a. The area of rectangle or volume of a rectangular block is equal to the product of the dimension
   b. The product of dimensions of a rectangle or rectangular block is equal to the area of the rectangle or volume of a rectangular block

      a. A is associated with factoring
      b. A is associated with multiplying
      c. Both A and B are associated with factoring
      d. Both A and B are associated with multiplying
4. The figure shows that (x +2) (x +3) is equal to
   a. $x^2 + 6x + 5$
   b. $x^2 + 5x + 6$
   c. $6x^2 + x + 1$
   d. $5x^2 + x + 1$



5. 

If these figures are assembled to form a rectangle, then the dimensions would be,
   a. (2x +1) (2x +2)
   b. (2x + 2) (x +2)
   c. (2x +1) (x + 4)
   d. (2x +4)(x +1)

APPENDIX C

TYPES OF LESSON

LESSON 1: Blocks representation

LESSON 2: Multiplication of Polynomials

LESSON 3: Factoring of Polynomials

LESSON 4: Cubic Polynomials