USE OF THE "X" TOOL TO IDENTIFY REGION THAT REPRESENTS GIVEN INEQUALITIES

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Abstract

The "X" tool consists of an X marked on a piece of tracing paper. It is a simple tool that students can easily make and use to identify the region that satisfies two linear inequalities. An action research on the use of the tool has been carried out on a mixed ability group of 43 Form 5 students and it has been found to be effective in assisting students to determine the region that satisfies two linear inequalities.

Introduction

Determining the region which satisfies two or more simultaneous inequalities is one of the learning outcomes associated with the Form Five topic 'Graphs of Functions' (Curriculum Development Centre, 2006). Evaluation of this learning outcome in the Sijil Pelajaran Malaysia examination often requires student to identify and shade the region that satisfies 3 inequalities. This region is usually bounded by 2 slanting lines and a vertical/ horizontal line.

Based on observation of students' performance on the above mentioned type of questions over the years it was noticed that a large number of Form 5 students, including some from the science classes, have difficulties deciding which region is to be shaded when they are given 2 or 3 inequalities. Despite the fact that students have been taught to choose a point on either side of a line to test whether it satisfies a given inequality many still were not able to give the correct answer in tests and examinations. This problem occurs when students are dealing with slanting lines although they do not have much problem with lines that are parallel to the axes. In other words, they have problem identifying the region for $y > mx + c$ or $y < mx + c$ where m is a non zero number but they do not have much problem identifying the region that satisfies $y > c$ or $y < c$ or $x > c$ or $x < c$.

A simple innovation called the "X" tool was created to solve this problem. An action research has been carried out to test the effectiveness of the "X" tool for identifying the region that represents two given inequalities.

Objectives of the Research

The main objectives of the research were to

(i) provide students with a concrete tool to figure out easily and quickly the region represented by two given inequalities in the form of $y > mx + c$ or $y < mx + c$ where m is a non zero number

(ii) boost students' confidence in answering SPM questions on the mentioned learning outcome..

Methodology and Procedures

Target Group

A total of 43 Form Five students of mixed abilities participated in this action research.

Timetable

Activity	Pre-test	Implementation (use of the "X" tool)	Post-test
Date	17 July 2008	21 July 2008	21 July 2008

Pretest

A Pre-test (Appendix I) of 10 questions each consisting of two inequalities of the form $y > mx + c$ or $y < mx + c$ where m is a non zero number was carried out on the target group. The students' task was to identify and shade the region represented by the two inequalities.

Analysis of Pre-Test Results

Table 1 shows students' performance in the pre-test. Only 15 students (35%) managed to obtain 9 or 10 correct answers while 24 students (56%) scored less than 7.

Table 1
Students' Performance in the Pre-Test

Number of correct answers	0	1	2	3	4	5	6	7	8	9	10
Number of students	0	3	3	3	5	5	3	2	4	3	12

Implementation

Making the Tool

An "X" tool is made by drawing two intersecting lines (one green and one red in colour) on a piece of tracing paper as shown in Figure 1. The top ends of the lines are marked with the inequality sign ">" while the lower ends of the lines are marked with the inequality sign "<".

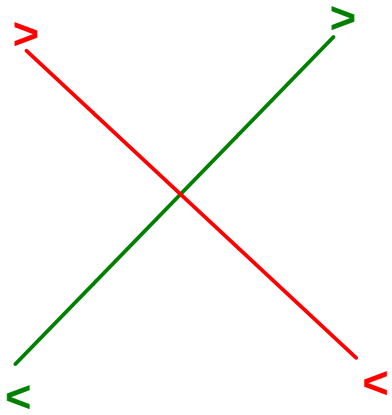


Figure 1. "X" Tool

How to Use the Tool

Example 1. The "X" tool is used to determine the region in Figure 2 which satisfies the inequalities $y \leq 3x + 10$ and $y \geq -x + 5$.

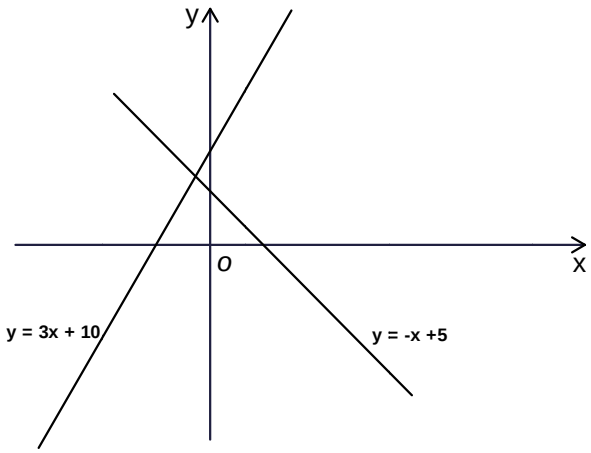


Figure 2. Sketch of $y = 3x + 10$ and $y = -x + 5$

One of the lines (red line in Figure 3) from the "X" tool is placed over the line $y = -x + 5$. To determine which side of the line $y = -x + 5$ represents the region $y > -x + 5$, look for the region where the inequality sign ">" in the other colour (green) is located.

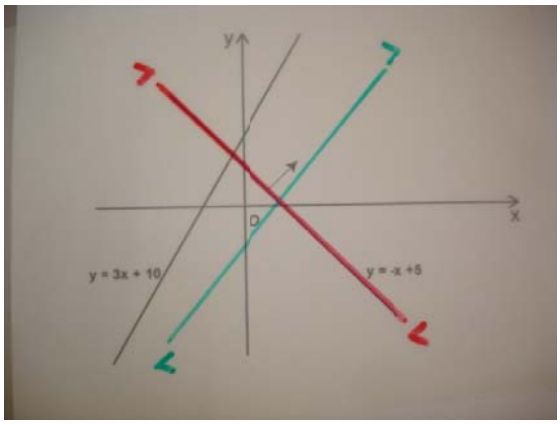


Figure 3. Use of the "X" Tool

The arrow in Figure 4 indicates the side of line y = -x +5 which satisfies the inequality y > −x +5.

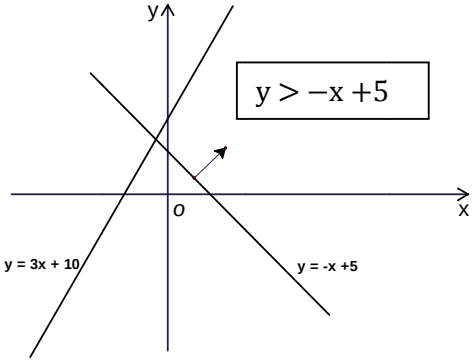


Figure 4. Side of line y = -x +5 which satisfies the inequality y > −x +5

Next, the green line from the "X" tool is placed over the line y = 3x + 10 as shown in Figure 5. To determine which side of the line y = 3x + 10 represents the region y < 3x + 10, look for the region where the inequality sign "<" in the other colour (red) is located.

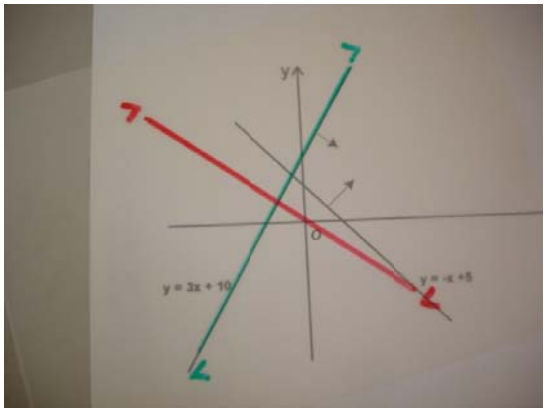


Figure 5. Locating the Region that Represents y < 3x + 10

The region which satisfies the inequalities $y < 3x + 10$ and $y > -x + 5$ is indicated by arrows in Figure 6.

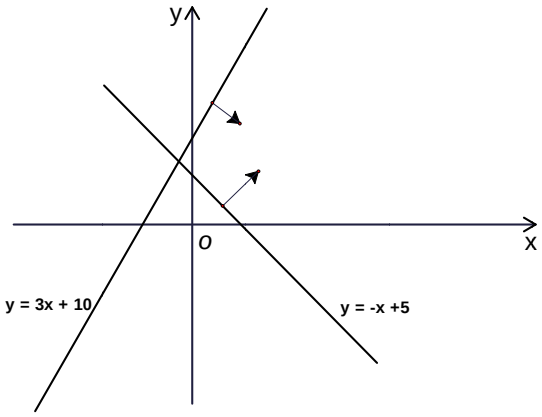


Figure 6. Region that satisfies the inequalities $y < 3x + 10$ and $y > -x + 5$ shown by arrows

The region which satisfies the inequalities $y \leq 3x + 10$ and $y \geq -x + 5$ is indicated by blue shaded lines in Figure 7.

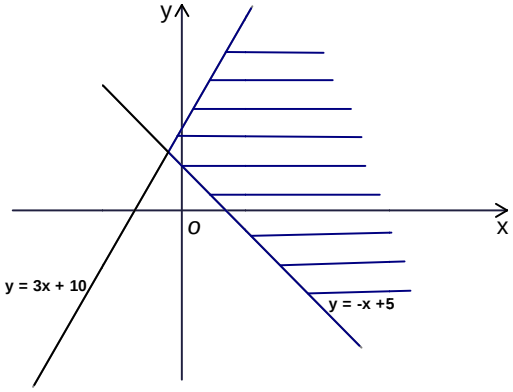


Figure 7. Region that satisfies the inequalities $y \leq 3x + 10$ and $y \geq -x + 5$ shown by blue lines

Example 2. Use of the "X" tool to identify the region in Figure 8 which satisfies the inequalities $y \leq 2x + 8$ and $y \geq x$.

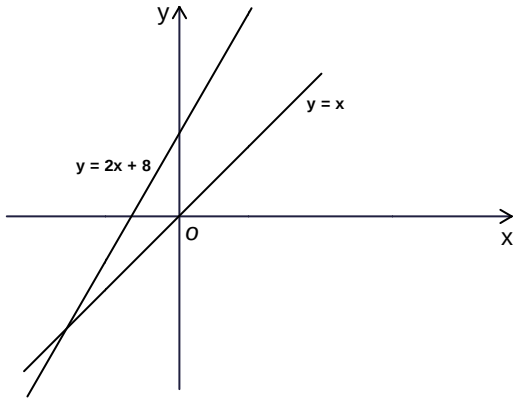


Figure 8. Sketch of Graph of y = 2x + 8 and y = x

One of the lines (green line in Figure 9) from the "X" tool is placed over the line y = x. To determine which side of the line y = x represents the region y > x, look for the region where the inequality sign ">" in the other colour (red) is located.

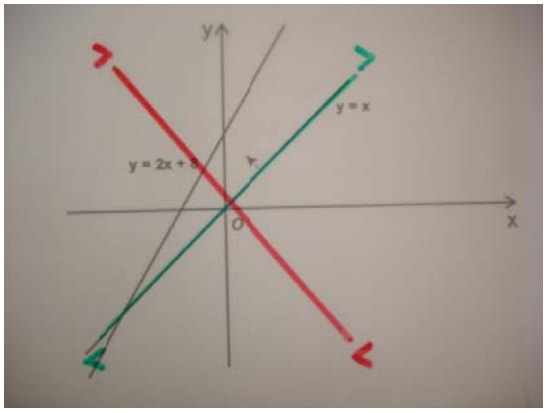


Figure 9. Locating the region y > x

The arrow in Figure 10 indicates the side of line y = x which satisfies the inequality y > x.

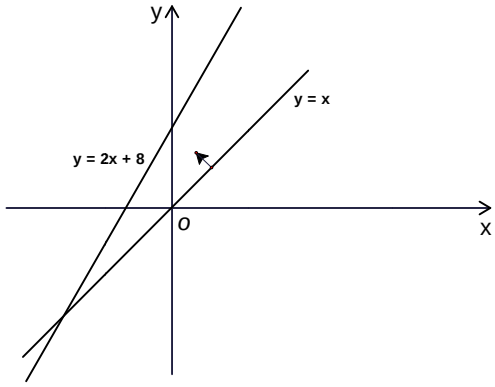


Figure 10. The arrow showing the side of y > x

Next, place the green line from the "X" tool over the line y = 2x + 8 as shown in Figure 11. To determine which side of the line y = 2x + 8 represents the region y < 2x + 8, look for the region where the inequality sign "<" in the other colour (red) is located.

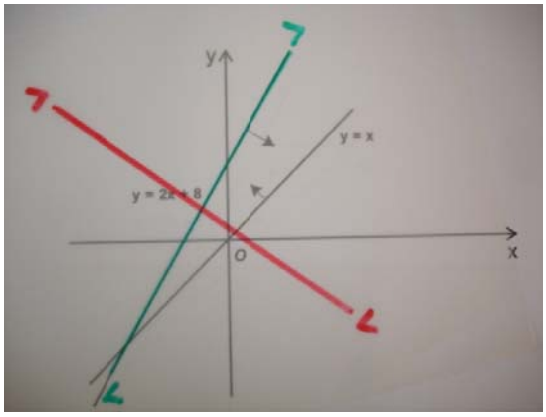


Figure11. Locating the region y < 2x + 8

The region which satisfies the inequalities y < 2x +8 and y > x is indicated by arrows in Figure 12.

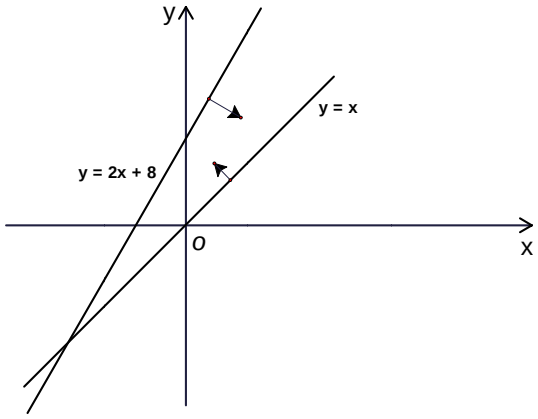


Figure 12. The arrows showing the region which satisfies the inequalities y < 2x +8 and y > x

The region which satisfies the inequalities y ≤ 2x +8 and y ≥ x is represented by blue shaded lines in Figure 13.

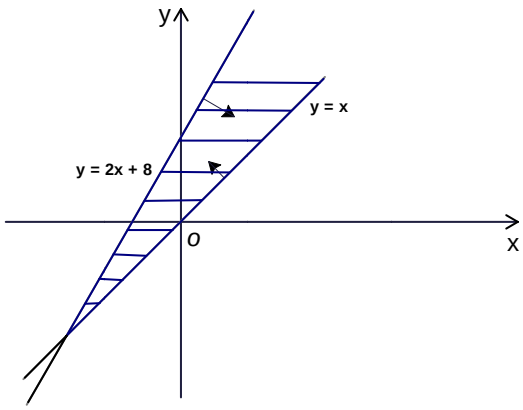


Figure 13. The blue lines indicates the region which satisfies the inequalities $y \leq 2x + 8$ and $y \geq x$

Practice

Students answered 5 questions by using the "X" tool to identify regions represented by given inequalities. Students presented their answers on the whiteboard.

Post-Test

Table 2 shows students' performance in the post-test. The post-test showed a remarkable improvement in the students' score with all the students scoring 7 and above and 35 students (88%) scoring 9 or 10.

Table 2
Students' Performance in the Post-Test

Number of correct answers	0	1	2	3	4	5	6	7	8	9	10
Number of students	0	0	0	0	0	0	0	1	4	6	32

Comparison Between Results of the Pre-Test and Post-Test

Figure 14 shows the pre-test and post-test results in the form of bar graphs. It shows a significant improvement in students' performance.

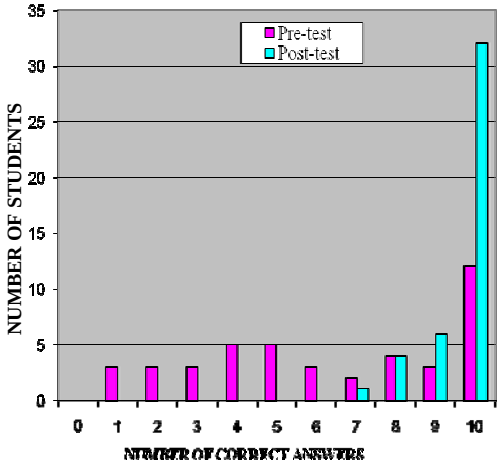


Figure 14. Comparison of Pre-Test and Post-Test Results

Cost of Research

The cost of tracing paper is less than RM 10.

Discussion

The idea for creating the "X" tool arose out of observations that the region for $y > mx + c$ is always (i) on the top right side of a line that slants to the left (negative gradient), and (ii) on the top left side of a line that slants to the right (positive gradient). These two observations are translated into a tool to help students identify the relevant regions very quickly for the same reason a calculator is being used to perform arithmetic calculations after students have learnt how to perform the calculations. However, it is important that this tool only be introduced after the concepts on region that satisfies inequalities have been taught and explored.

Some students realized after a while that the region $y > mx + c$ is always represented by the top part of the "X" while the region $y < mx + c$ is always represented by the bottom part of the "X". In the case of a line slanting to the left, the region $y > mx + c$ is always on the top right. Similarly for a line slanting to the right the region $y > mx + c$ is always on the top left.

Reflection

Students were excited and enjoyed the activity of using the "X" tool that they had made from tracing paper to find the correct answer. Students expressed their view, that the use of the tool made it easier to figure out the answer very quickly but then asked, if they would be given tracing paper in the SPM examination. It was during this part of the discussion I came up

with the idea of making the "X" tool by tearing a small square (3 cm by 3 cm) from a corner of a piece of paper. Students can make a small "X" tool in this manner during the examination and place it over a given diagram to determine the region to be shaded. (refer to Figure 15). The advantage of having a much smaller "X" tool is that it does not cover up the lines in the diagram.

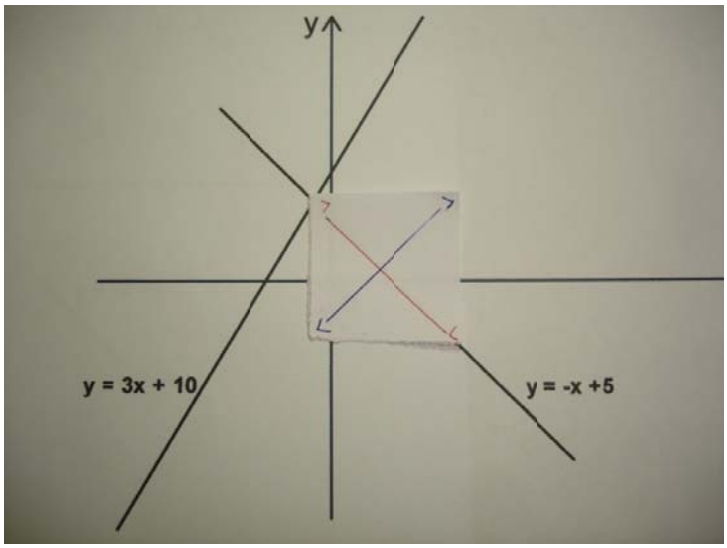


Figure 15. Small "X" Tool

I felt a sense of satisfaction as the result of the students' excitement and involvement, as well as their confidence in answering the questions. It was also observed that rotation of the "X" tool by 180° does not change the position of the ">" and "<" signs. Appendix II shows five photographs of the actual use of the "X" tool by the students.

Suggestion for Further Research

The use of this tool is not limited to region involving 2 slanting lines. It can be applied to identify region involving more than 2 slanting lines.

Further action research can be carried out on the use of this tool in answering SPM linear programming questions in Additional mathematics. Students would need to rearrange the inequalities of $ax + by > k$ or $ax + by < k$ so that either (i) the inequalities are in the form $y > mx + c$ or $y < mx + c$,or (ii) the coefficient of y is positive.

Conclusion

The use of the "X" tool has made significant impact on students' ability to identify the region that satisfies 2 given linear inequalities in the form of $y > mx + c$ or $y < mx + c$ where m is a non zero number . By using the tool, students were able to identify the region very quickly and accurately. The "X" tool is particularly helpful to the weaker students.

Reference

Curriculum Development Centre (2006). *Integrated Curriculum for Secondary Schools: Curriculum Specifications Mathematics Form 5*. Putrajaya, Malaysia: CDC, MOE.

Appendix 1

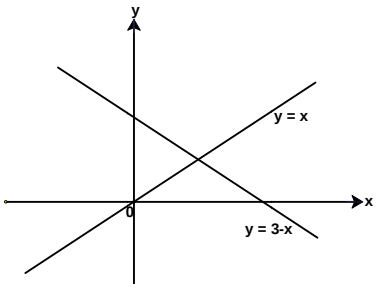
Pre-Test

NAME: …….. CLASS:……

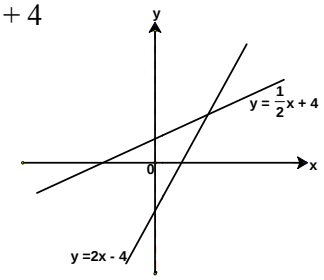
REGION REPRESENTING INEQUALITIES IN TWO VARIABLES

On each of the following diagrams, shade the region which represents the respective inequalities.

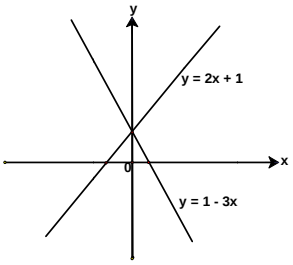
1. $y \geq 3 - x$ and $y \leq x$



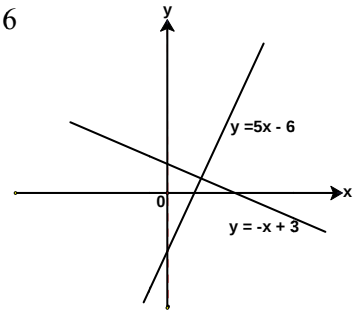
2. $y \geq 2x - 4$ and $y \leq \frac{1}{2}x + 4$



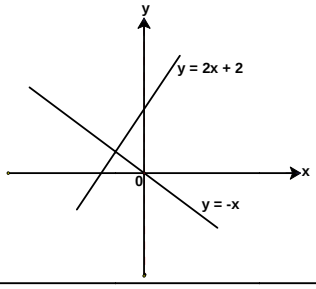
3. $y \leq 2x + 1$ and $y \leq 1 - 3x$



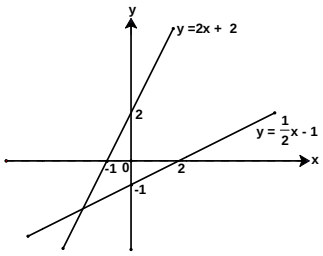
4. $y \leq -x + 3$ and $y \leq 5x - 6$



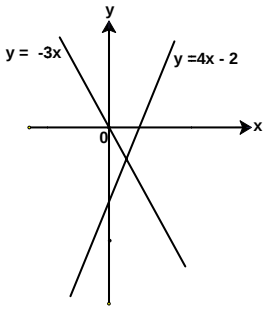
5. $y \geq 2x + 2$ and $y \geq -x$



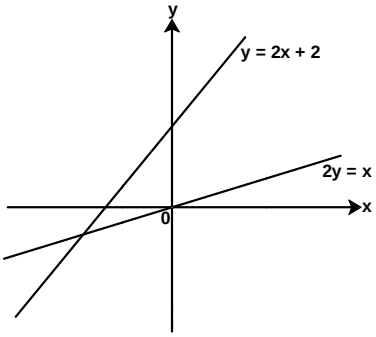
6. $y \leq 2x + 2$ and $y \geq \frac{1}{2}x - 1$



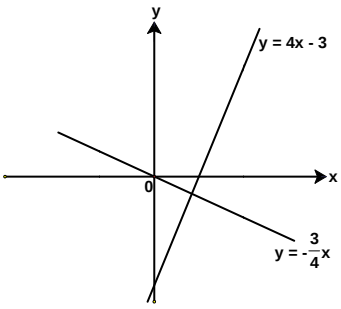
7. $y \leq -3x$ and $y \leq 4x - 2$



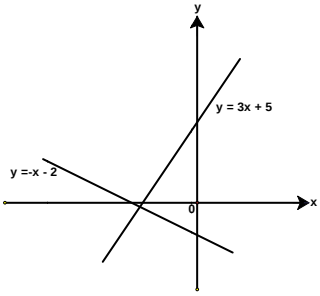
8. $y \leq 2x + 2$ and $2y \geq x$



9. $y \geq 4x - 3$ and $y \geq -\frac{3}{4}x$



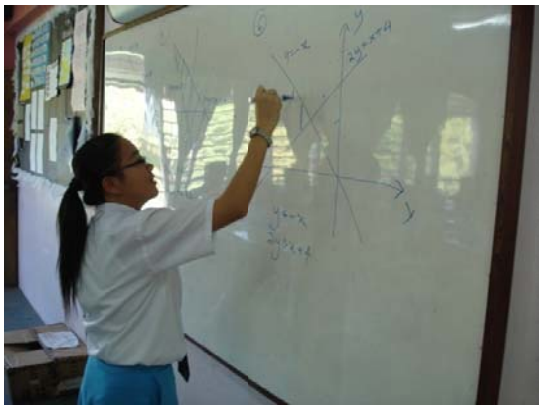
10. $y \geq 3x + 5$ and $y \geq -x - 2$



End of Question Paper

Appendix II

Student using the tool she made



Student presenting her answer on the whiteboard

Student using the tool to answer post-test

Student using the tool to check her answer

Students answering post-test