

## TEACHING NUMERACY THROUGH REKENREK

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### Abstract

*One of the more important tasks for primary mathematics teachers is to help young children develop powerful mathematical understandings of numbers, their meanings, their relationships to one another, and how we operate with them. The rekenrek which is developed by mathematics education researchers in the Netherlands combines various strengths of other manipulatives (e.g., number lines, base-10 blocks, counters, etc.) in one accessible tool. It is a powerful tool for teaching a range of strategies for addition and subtraction which can be extended to multiplication and division as well as enhancing pupils understanding and applying the properties of operations. However, rekenrek is new to most countries in this Southeast Asia region. This study in the form of a workshop introduced Rekenrek to a group of mathematics teachers as one of the topics of their in-service course. The objective is to explore teachers' perception in using rekenrek as mathematics manipulative during their lesson after they have undergone training and to determine the trained teachers' perspective on the effectiveness of using rekenrek to develop pupils' number sense. The data were collected through a questionnaire, observations made by the authors as well as analysis on their completed tasks which is part of the workshop activities. The study revealed the many positive perceptions the teachers had on the use of rekenrek in schools. Among them are the ability to draw out the many different strategies from the pupils, promoting exploration, subitisation, decomposition and automaticity. The study also reveals that it is quite easy for the teachers to introduce its use as well as to make rekenrek in the mathematics classrooms.*

### Introduction

Brownell's Meaning Theory is based on the postulate that children must understand what they are learning if learning is to be permanent (Kennedy, 1980). Therefore, for learning to occur, the 21<sup>st</sup> century mathematics teachers need to move beyond their role of knowledge providers to opportunity creators. They carry the responsibility of creating classroom instructions that engage pupils to discover mathematics on their own. By designing meaningful activities using manipulative materials (Kennedy, 1980), teachers pedagogically are able to guide pupils to communicate mathematically and express their views about the mathematics that they are learning (Kennedy & Tipps, 1990). Through this form of rich mathematics discourse, they are led to investigate and build their own understanding of the mathematical concepts (Kennedy, 1980).

Learning new mathematical concepts or topics at any grade level ought to begin at a concrete stage (Gardella, 2009). According to Kennedy (1980), the sequence of introducing new topics to pupils should move from a concrete-manipulative mode to the semi concrete pictorial mode before advancing to the abstract mode. Difficulties in learning occur when the

last two modes receive great emphasis or when pupils are hurriedly led to the abstract mode. This is because children exhibit different degrees of mathematical understanding and their ability to grasp new mathematical concepts vary. Therefore, they need sufficient opportunities to explore the concrete representations. Once they have grasped the concrete representations, they are more able to operate on the less concrete modes.

Understanding numbers is one very important and most fundamental concept in mathematics. Pupils' mathematical learning track begins with the understanding of number meanings and the association between number words and the corresponding objects. Without proper understanding of number sense, pupils' learning of other content areas like performing operation with numbers is stunted. As such, a teacher' greatest challenge is perhaps guiding pupils to develop an in-depth understanding of number sense. One alternative being proposed in this paper is the use of Rekenrek.

### **Mathematical Learning and Counting among Children**

Children's mathematical learning progresses from Piaget's sensorimotor stage to the preoperational stage before advancing to the concrete operations and formal operations stages. However, their mathematical understanding starts to form only during the preoperational stage of age range between two to seven years. At the age of seven to eleven years when they are at the concrete operations stage, they need to be constantly in contact with objects that they can manipulate in order to develop their number sense. At the end of this stage, they will be able to operate efficiently without concrete materials and transit to the final stage of formal operations after the age of eleven. At this stage, they will be able to think analytically before advancing to adult thinking (Kennedy, 1980).

Counting is one of the fundamental skills in learning mathematics that requires a wide range of complex skills. Biggs and Sutton (1983) deduced that counting involves a variety of experiences like sorting and classifying, matching, learning number names in a sequence, matching number names sequenced to the number of objects, starting counting with different object, recognising a set of three (developed to four and five) objects without counting and acquiring the abstract concept of sets of specific number.

When beginning mathematics instruction among children in Grade One, teachers need to be aware that some children may not be ready for number work as they are still at the preoperational stage. Since some children can count to ten or higher through rote learning, they should not be misdiagnosed as ready to learn mathematics as found in a study conducted in University of Missouri. The findings indicated that children who demonstrated counting skills in preschool obtained the highest scores in Grade One than preschoolers who could only recite (Rasicot, 2012). This is because children are able to recite number sequence without fully understanding the meanings of number (Kennedy, 1980) as reciting involves memorising number sentences while counting engages children in a cognitive activity of one to one mapping of number words to the number of objects (Rasicot, 2012).

According to Copeland (1974), the ability to memorise sounds in a sequence is the first level of counting known as rote counting and should not be misconstrued as having acquired the ability to count. Understanding of numbers can only be established when children can demonstrate a one-to-one mapping of the number names to the objects being counted, known as rational counting (second level of counting). Matching sets of manipulative materials like papers to pencils or dresses to dolls is a sign of having acquired rational counting. When these three-dimensional objects are complemented by the proper use of workbooks that allow children to draw lines between two-dimensional pictures in the book, it signifies children's progressive development from the concrete stage to the abstract stage, which supports

mathematical learning. Even though the teacher designs the activities with manipulative at the concrete stage to help the child learn numbers, only the child can sort out the number in the abstract mode. Since the child is left on his own to complete the transition to the abstract mode, teachers need to prepare a conducive learning environment that is richly embedded with manipulative materials to ease his progression. Until this is done, mathematics understanding cannot possibly take place.

The final level of counting, known, as conservation of numbers requires children to determine the equivalence of two sets of objects. Once children at the concrete stage reach this level of counting, they are ready to learn addition and subtraction (Copeland, 1974).

### **Learning of Addition/Subtraction**

Counting numbers is a content area that can be used to solve almost any problem related to discrete objects. When pupils demonstrate the ability to count with proper understanding, it is an indication of a measure of readiness to learn subsequent mathematical tasks like addition before advancing to subtraction. The importance of having a deep understanding of addition cannot be undermined as children tend to use indirect addition to solve subtraction problems (Nunes, Bryant, Evans, Bell, & Barros, 2012). However, this method appeals to children only when they are provided with external aids like empty number lines (Torbeyns, Smedt, Stassens, Ghesquiere, & Verschaffel, 2009) or manipulative materials. In addition, a good understanding of the concepts related to addition will also guide their learning of multiplication as repeated addition (Kennedy & Tipps, 1990). Their ability to work efficiently with these four operations of addition, subtraction, multiplication and division is fortified when the principle of inversion (addition is the inverse of subtraction and multiplication is the inverse of division) is introduced. When pupils adopt additive composition (a number is composed of other numbers and can be decomposed into other numbers), they are actually applying the principle of inversion (Bryant, 2011). As such, for children to possess the knowledge of number sense, they need to display a profound understanding of counting which advances to addition, and the three arithmetic operations.

Learning to add and subtract springs from children's early childhood playing experience. By engaging in meaningful play-like activities, they learn to explore mathematical concepts like height and distance, which sets a strong mathematical foundation. Manipulative like dolls or beansticks can help to develop the concept of addition as children explore from single digit to bigger numbers. Allowing children to participate sufficiently with manipulative at the early stages of introducing addition and subtraction, and delaying the abstract mode of paper-pencil work can greatly curb the habit of 'finger counting' among children. Once they have grasped the concrete representation, they can be led slowly to the semi concrete pictorial mode by using a number line before paper-pencil work. Among some children, when working with challenging tasks, they occasionally tend to revert to the concrete representation (Kennedy & Tipps, 1990).

### **Manipulative Materials**

The use of manipulative materials is pivotal in children's mathematical maturation and mathematics achievement, especially when they are at the concrete stage (Gardella, 2009). Manipulative materials cover a whole range of learning tools (Biggs & Sutton, 1983) like numeral cards (Kennedy, 1980), physical objects like cups and saucers, dice and dominoes, coloured objects like beads (Biggs & Sutton, 1983), Cuisenaire rods, Dienes blocks, pattern blocks, diagrams, pictures (Gardella, 2009), measuring instruments, math box, play money,

tangrams, geometric models and Geoboard (Kennedy & Tipps, 1990). They are essentially teaching-learning aids that allow pupils to ‘see’ the mathematical concepts in different ways which will enhance their mathematical understanding.

Using objects that are present in the children’s environment like rocks and toys can help in developing their basic skills which creates the foundation to the more difficult concepts learnt later (Truesdell, 2012). Appropriately, selected manipulative materials introduced during a well-planned phase of lesson can also promote interest and motivate problem-solving skills. They help to connect the real object with the abstract mathematics (Kennedy & Tipps, 1990). Pupils are able to bridge their initial phase of abstraction to their advanced thinking of symbolism as a representation when they interact with these physical objects, (Gardella, 2009). Thus, they learn better when many suitable manipulative are adopted in classroom activities (Kennedy & Tipps, 1990).

Inadequate exposure to concrete representations using manipulative materials and, excessive or complete reliance on textbooks or workbooks deny them from any opportunity to interact with manipulative. This form of teaching strategy need to be avoided as it slows down the development of their numeracy skills (Kennedy, 1980). Instead, it is recommended to tap on children’s rich experience with manipulative materials as it allows them to establish the physical representation of their mathematical ideas into symbols that are essential in mathematical communication (Gardella, 2009).

### **Rekenrek**

Freudenthal Institute in the Netherlands developed rekenrek, also known as an arithmetic rack (Fosnot & Uittenbogaard, 2007), calculating frame or counting rack (INQUIRE, 2008). Its design was spurred by the informal early learning strategies that children adopt while studying numbers and, the operations of addition and subtraction. According to Frykholm (2008),

*The Rekenrek provides a visual model that encourages young learners to build numbers in groups of five and ten, to use doubling and halving strategies, and to count-on from known relationships to solve addition and subtraction problems. (p. 1)*

The fundamental principles behind the construction of rekenrek is the number line, counters and base-10 models that have long been used as a common teaching practice for developing pupils’ understanding of number sense. It consists of two rows with ten beads strung into each row. Each row bears two groups of five beads. One group has five red beads while the second group has five white beads. Since rekenrek has ten beads in each row and for each row, there are two groups of five coloured beads, it invites pupils to think in groups of five and ten. For pupils who need extra support, the construction of rekenrek can be modified into rows of five beads each or ten beads each or even to twenty beads each row for the advanced learners (Frykholm, 2008).

### **The Mathematics of Rekenrek**

Rekenrek may look like an abacus since it also teaches pupils number sense strategies. However, the mathematical principles behind its design stretches beyond that of an abacus. It is neither based on place value nor base-ten structure like an abacus. It supports the learning of important mathematical skills like automaticity, subitisation and flexibility. Automaticity is the

ability to automatically give answers within seconds by inspecting the relationships among operations without having to recall them or rely on counting. Subitisation is the ability to recognise the number of objects in a group without having the need to count. The five beads structure of the rekenrek allows pupils to easily 'see' the quantity five as a whole without having to count (INQUIRE, 2008). This mathematical innovation adds the advantage of breaking away from the convention design of creating mathematical manipulative based on mathematical principles that are apparent to adults but not to the children (Fosnot & Uittenbogaard, 2007). Flexibility focuses on pupils' ability to understand the strategies behind the operations of adding and subtracting (INQUIRE, 2008).

The design of rekenrek is based on some fundamental mathematical properties of cardinality, subitising and decomposition (part-part-whole). Cardinality refers to correspondence between the number of objects in a set and the numeral denoted for the grouping while subitising is recognising a group within a bigger group without having to count. Another underlying principle is decomposition of numbers, which is determining the individual parts that compose the whole.

Since children can 'see' the numbers that are represented by the beads, rekenrek conveniently 'shows' that a number is a combination of two or more numbers. An important feature that aids in the recognition of quantities is the use of two different colours for each set of five beads. This gives an added advantage for the children who are beginning to learn numbers. With the coloured groups of five beads strung in a row of ten beads, pupils can also see that 7 is a group of five beads (first row) and another two more beads (in the first row). Rekenrek allows pupils to explore creatively as pupils can also subitise 7 as a group of five beads (first row) and another two more beads (in the second row). The mathematical thinking of visualising numbers within other numbers is instrumental and provides the prerequisite knowledge for addition and subtraction. This visual mode greatly helps in developing multiplication in the later stages when 7 is decomposed to 'three groups of two and one more' or 'two groups of three and one more' or 'four groups of two and less one' or even 'two groups of four and less one'. Teachers can use this form of investigation to guide pupils to think creatively and communicate the mathematics that the children themselves 'see'. Similarly too with 13 which at the beginning stages of learning is seen as 'a group of 10 beads (in the first row) and 3 more (in the second row)', which can be extended with further exploration as the lesson advances (Frykholm, 2008).

In a study conducted by Tournaki, Bae and Kerekes (2008) among children with learning disabilities, rekenrek was found to be effective in enhancing pupils' learning of addition and subtraction. These pupils were divided into three groups. Group 1 received instruction with the use of rekenrek, Group 2 received instruction without using rekenrek while group 3 received no instruction. The high R square of .71 indicated a high percentage of 71% of variance was explained by the usage of rekenrek. They concluded that pupils' development of number sense was supported by pupils' awareness of relating the five-base structure of rekenrek to their five toes and five fingers. In addition, the use of rekenrek also helped these pupils to clearly explain the mathematical operations as they were 'seeing' the mathematics that they were 'doing'. Moving the beads of ten within the five structure of different coloured beads provided them the opportunity to explore the base ten structure and base five structure simultaneously.

## **Theoretical Framework**

### **Realistic Mathematics Education**

Realistic Mathematics Education (RME) looks at mathematics as a human activity, where learners mathematise the subject matter, which is drawn from their experience and real context. RME views pupils as actively learning and re-inventing mathematical knowledge, instead of being passive learners who receive input from teachers. This is opposed to learning mathematics as a presentation of facts that are readily accepted through a procedure-bound experience with pupils mastering the mathematical concepts through exercises. Through guided re-invention, learners discover mathematics by experiencing horizontal and vertical mathematisation. Horizontal mathematisation occurs when learners depend on their informal ways through their experience to describe and solve the contextual problems. Vertical mathematisation occurs when these ways direct them to problem solving by identifying the appropriate algorithm or by using the mathematical language (Barnes, 2005). Heuvel-Panhuizen (2001) differentiates the two as the system within which they function. Horizontal mathematisation moves from the mathematical problem to the real world, while vertical mathematisation moves within the mathematical system of symbols.

In RME, the contextual problems and real-life situations provide a guided setting for learners to actively re-invent and apply the mathematical knowledge and concepts through their mathematisation. These contexts eventually will become too general and become a ‘model’ for pupils to solve other related problems and eventually become a scaffolding to generate and learn the mathematical knowledge. As such, RME teaching methods creates opportunities for learners to share their mathematical experience, while they are actively ‘doing’ the mathematics.

### **Research Rationale and Research Objective**

Teachers’ role in every aspect of education is significant and affects pupils’ learning. A teacher’s crucial role lies in determining how they can deliver the mathematical concepts to their pupils in a way that their pupils learn by self-discovery and not through presentation of ready-made facts that they are expected to readily accept. As such, teachers’ role in guiding pupils’ learning begins with them having an in-depth mathematical knowledge that they plan to bring to the classroom and use a mathematical manipulative to develop that mathematical concept. An issue in the using of manipulative is that if teachers are uncommitted in using it or do not believe in its effective use as a teaching aid, they will less likely to be able to use it to develop their lessons, despite having in their possession an efficiently research-based designed mathematical manipulative. On the other end, committed teachers too will fail to deliver a good lesson if training on the effective use of manipulative is not provided. Therefore, there is a need to ensure that teachers are trained be effective users of the mathematical manipulative (Marshall & Paul, 2008). As such, this research is a modest attempt to explore teachers’ perception in incorporating rekenrek as mathematics manipulative in their classroom after receiving training and carries the objective of determining the trained teachers’ perspective on using rekenrek to develop pupils’ number sense and the extent of its usage as a mathematics manipulative.

## Methodology

A workshop of two hours was conducted by one of the authors. In the first hour, the author examined together with the participants how rekenrek, which is a simple but powerful manipulative can help pupils to develop higher order thinking skills and promote mathematical understanding. Specifically, they explored the rationale for the use of rekenrek, the mathematics of rekenrek, and hands-on activities that showed how rekenrek can improve pupils' thinking skills; understanding and proficiency with addition and subtraction, number sense, and base-ten system.

In the first session of the workshop, the participants were guided to construct their own rekenrek from a small cardboard, string and 20 beads of two different colours. This self-built rekenrek would later be used as part of the training activities. Among the activities introduced were how rekenrek could be used to guide students to enhance and develop their mathematical understanding in the concept of cardinality, automaticity, subitisation, grouping of fives and tens, doubling, almost double, decomposition and addition. In the second half of the workshop, participants were required to design activities using the rekenrek by adapting these activities shared but extended to subtraction with the focus on developing (i) an understanding of part-part-whole relationship in number problems, (ii) a relational understanding of the equal sign, and (iii) confidence and comfort with 'missing subtrahend' problems.

At the end of the workshop, a qualitative questionnaire was given to them which required them to write out their responses to six questions related to the rekenrek. The questions in the questionnaire were adapted from Marshal and Paul (2008). The purpose of selecting these questions were to invoke the participants' views and opinion in using rekenrek as a newly learnt teaching aid to teach simple mathematical operations as opposed to other conventional teaching tools that were used in the past. This is because the authors hoped to investigate the comparability of rekenrek against computer aided tools that were used to teach these operations (if any). The questions that outlined the questionnaire are as follows:

- 1) Explain how you use rekenrek to introduce and teach addition/subtraction to your pupils?
- 2) What are the advantages (if any) of using rekenrek to teach addition/subtraction in your classroom?
- 3) What are the disadvantages (if any) of using rekenrek to teach addition/subtraction in your classroom?
- 4) State the obstacles that you might be experiencing when using rekenrek in your classroom?
- 5) When using rekenrek will the pupils' experience be based on (a) Teacher direction or (b) Questions arising from self-discovery

These written responses were then analysed to determine their initial reaction to using rekenrek and possibly, determine the extent of possibly using it as a teaching aid potential for future exploration. It is also desired that these findings will be able to shed some light on shaping and designing future training workshops on rekenrek.

## Sample

There were 42 participants of this study with 35 female participants and 7 male participants. They were participants of a one-week in-country course provided by the authors' centre as an in-service course. The in-country courses are offered by the authors' centre to its member countries for training their educators in their own country upon request by the Ministry of Education of the member country. One of the authors was their course supervisor as well as

the facilitator for the workshop on rekenrek. The 42 participants were all elementary teachers from all over the Philippines. All of them had a Bachelor degree and five had a Masters degree. Table 1 shows the breakdown of their age category.

Table 1

*Participants' Age Category*

Age (years)	Below 24	25-29	30-34	35-39	40-44	Above 45	Total
Number	2	6	6	7	8	13	42
Percentage (%)	5	14	14	17	19	31	100

From Table 1, it can be inferred that majority (31%) of them were experienced mathematics teachers who were above 45 years old and 81% of these participants were above 30 years old. This suggests that they have had many years of teaching these mathematics operations to elementary pupils and as such, would have resorted to using a variety of teaching methods and utilising multiple teaching aids to enhance pupils' learning. As such, they are a rich source of information as they are able to compare these teaching aids to rekenrek.

### Results

The results will be discussed in two parts. The first section will focus on the activities that they had designed and the participants' reactions while designing them. The second part will explore the participants' written responses to the five items enquired in the questionnaire. Their comments and the discussion will be presented according to these items.

#### Part I

The participants during the second half of the workshop created activities that they believed could be used to guide pupils to mathematise the arithmetic operations using rekenrek. A sample of some of the activities are as shown in Figures 1, 2, 3 and 4, in addition to their observable reaction while developing the activities in their groups.

#### *Observation:*

All the participants disclosed that rekenrek was a new manipulative to them but it was similar to those counting racks they had used. They showed excitement and were amazed to discover it to be different from the usual 10 rows counting rack that they had used in the past. They related that rekenrek looked simple and yet, was a well thought out tool to help and guide the pupils by encouraging informal strategies for addition and subtraction through its use of two different sets of coloured beads that were grouped in fives.

The development of activities using rekenrek for learning subtraction did not pose any problem. Below are a set of activities created by a group:

“Let's make 8. I start with 9. How many less?”  
 “Let's make 4. I start with 7. How many less?”  
 “Let's make 6. I start with 8. How many less?”

*Figure 1. Activity A.*



- Push some beads to the left. Cover the remaining beads.
- Ask pupils: “How many beads do you see on the top row?”
- Ask: “How many beads **are covered (top row)**? Use the word “less” in your answer.”
- Listen for answers like the following:
  - “7 and 1 less is 6. I counted up to 10 ... 7, 8, 9, 10. 4 are covered.”
  - “I know that  $6 + 4 = 10$ . I see 6, so 4 less.”
  - “I know that there are 10 beads in a row. I see 6, so 4 less.”
  - “I know that there are 5 red and 5 white on each row. I only see one white, so there must be 4 less.”
- Next... move to both rows of beads.

*Figure 2. Activity B.*

What belongs in the box?

$$8 - \square = 5$$

*Figure 3. Activity C.*

What belongs in the box?

$$8 - 4 = \square - 5$$

*Figure 4. Activity D.*

## Part II

### 1) Explain how you use rekenrek to introduce and teach addition/subtraction to your pupils?

Majority of the participants commented that they found the base-five structure very helpful in teaching addition/subtraction and as such, rekenrek was more suitable than base-ten structures. One participant felt that pupils should be guided into building their own rekenrek as they would discover the mathematical properties that supported its design and would be able to associate better to the mathematics as they have had experienced. Using a ready-made rekenrek would defeat this purpose.

One participant revealed his enthusiasm in immediately using rekenrek as he believed that even slow learners could benefit greatly as they would be able to visualise the operations and associate the two sets of five beads to their five fingers and five toes. He also added that by using rekenrek consistently, slow learners would eventually master through ‘drill and practice’. This supports the notion of RME that contextual problems will eventually become a model for pupils and will support their mathematical learning. The rekenrek was also believed to be very beneficial for fast learners as one participant commented that it would be very suitable to develop the idea of ‘doubling’.

Even though many participants believed that rekenrek was appropriate for building pupils’ understanding of arithmetic operations and for developing higher thinking skills, there was still a misconception about the use of rekenrek as only a counting tool.

**2) What are the advantages (if any) of using rekenrek to teach addition/subtraction in your classroom?**

Majority of the participants was in the opinion that rekenrek provided visual representation of structures five and ten, and as such contributed significantly to the learning of addition and subtraction. One participant felt that the reason behind this was that rekenrek was tangible and therefore, the concrete representation that rekenrek offered helped to teach the abstract arithmetic operations, while another participant added that colour was another feature of rekenrek that encouraged pupils' learning. They also believed that long term and consistent use will lead to pupils' mastery of these operations. Some participants felt that using rekenrek promoted fun learning as it deviated from the traditional 'paper-pencil' method, and that it was less expensive and very convenient to be used. A small group claimed that by using rekenrek, pupils need not depend on fingers to count as they would be able to obtain the number of objects by seeing in groups. In addition, they would also be able to quickly obtain answers since no counting was involved. They also believed that it helped to develop pupils' mental calculation.

**3) What are the disadvantages (if any) of using rekenrek to teach addition/subtraction in your classroom?**

Many responded that they were no disadvantages in using rekenrek to teach addition/subtraction. The minority who responded claimed that rekenrek looked too attractive and therefore, could distract pupils. They might treat it as a 'toy' and end up 'playing' with it. As such, they felt that teachers must know how to use it effectively so that it does not lose its essence as a mathematics manipulative. A participant's comment, "Teacher is the manager and it is how she/he manages every single detail in the activity" resonates the importance of teachers becoming efficient users of mathematics manipulative like rekenrek. Another disadvantage cited was the time factor. Participants felt like rekenrek just like any other mathematics manipulative would consume time.

**4) State the obstacles that you might be experiencing when using rekenrek in your classroom?**

Many participants provided responses similar to item 3. They cited time as an obstacle, in addition to a minority group who felt that rekenrek was only suitable for young children. They believed that it was neither suitable for children from grades 4 to 6 nor could it be used to teach large numbers. One participant commented that with an influx of high technology gadgets available in the market, rekenrek will bore pupils. Interestingly, one participant commented that rekenrek cannot be used to teach multiplication.

**5) When using, rekenrek will the pupils' experience be based on (1) Teacher direction or (2) Questions arising from self-discovery**

All the participants responded that when using rekenrek, pupils' experience will be based on questions that arose from self-discovery and not teacher directed.

### **Discussion and Conclusion**

Based on the activities that the participants managed to create, they seem to display the ability to aptly design activities that invoke pupils' discovery of the mathematical concept of subtraction/addition in their own different ways with the use of rekenrek. This is consistent with Frykholm (2008) that these activities can be in the form of exploration to guide the pupils to think creatively. A close analysis of the activities designed seem to indicate that each activity

by itself is able to withdraw many different strategies from the pupils and as such, that one problem posed in the activity allowed pupils to explore varied number of solutions. It is these types of activities that can promote better mathematical understanding in the students when they utilise the desired strategies such as subitising, decomposition, doubling and almost double highlighted by Fosnot and Uittenbogaard (2007) and Frykholm (2008). The activities designed by the participants could indicate that the participants have the perception as advocated by Kennedy (1980) that by engaging the pupils to discover mathematics it will lead them to investigate and build their own understanding of mathematical concepts. Activities such as these also support the notion by Biggs and Sutton (1983) that counting involves a variety of experiences and requires a wide range of complex skills.

In addition, as exhibited in Figures 1 and 2, the guidance in the form of questions posed elicits pupils to share their experiences in arriving to their solutions, which we believe will lead to pupils' development of the mathematical language. It is evident that through these activities and the questions posed while using rekenrek, pupils are able to establish the physical representation of their ideas into abstract forms, which creates the appropriate learning environment for pupils to engage in an informal and formal mathematics discourse (Gardella, 2009; Kenny, 1980).

Even though majority of the participants found the design of rekenrek could develop higher mathematical thinking, there was still a participant who reduced its efficient use to merely serve as a counting tool and another falsely claiming that its usage cannot be extended to teach multiplication. This perception in terms of consistency oppose the trend suggested in the literature by researchers such as Frykholm (2008) and Tournaki et al. (2008). This suggests that during the training sessions, it is very critical to introduce the primary purpose of using rekenrek and to differentiate it from other manipulative that are similar to it. By doing so, it might alleviate confusion. As commented by a participant, teachers ought to be efficient managers of the manipulative so that it is well-used in class. In addition, the training session should also accommodate extended exploration among the participants so that they also able to discover on their own the additional usefulness of rekenrek.

From their reactions while developing the activities and verbal responses, these participants seem to be impressed by the mathematical properties behind the design of rekenrek, especially with two sets of five beads that emphasise the five-base structure within the ten-base structure. This could suggest that the participants perceived similarly with the findings of Frykholm (2008) that rekenrek provides a visual model for the pupils to build in groups of five to solve counting problems. This unique physical property seems to deviate from the mathematical manipulative like the base ten-structure that displays ten-base structure units but not the five-base structure. Colour is also an important feature of rekenrek that supported pupils' learning of the addition and subtraction, as they were able to subitise and also automate the answers quickly without having to count. Also, this unique way of structuring the beads in different colours into groups of five not only promote the understanding of the concept of grouping, decomposing and doubling but also promote the interest in students to use it as a tool. Accordingly, many participants reported rekenrek as attractive.

Very few participants who commented on the disadvantages and obstacles of using rekenrek in the classroom cited time as a major hindrance. In addition, the suitability of using rekenrek among older children and to teach bigger digits was questioned. This evidence is in contrast to Gardella (2009) as he recommends the use of manipulative materials to allow pupils to establish the physical representation of their mathematical ideas. At this point, it is important to be reminded that rekenrek or any other manipulative has its purpose. The purpose of rekenrek is to expound and develop pupils' development of number sense and as such, it is not useful among pupils who have mastered it. There is no age limit imposed on its users or constraints on the number of digits in using rekenrek. The issue, however, remains that rekenrek is not

suitable among older children or while teaching addition/subtraction of bigger digits as it has outlived its intended purpose. It is the younger children whose learning needs to be catapulted from the concrete stage as Gardella (2009) reiterated. Older children who at the formal operation stage or at then end of the concrete stage have already established these abstract mathematical concepts and as such, can wean away from tangible manipulative like rekenrek.

All the participants agreed that rekenrek was an interesting teaching aid that could be used in teaching mathematics among younger children as it allowed them to explore, discover and develop number sense. This study also shows that creating a simple rekenrek for use in teaching the various counting strategies is easy, effective and cheap. They were very optimistic and enthusiastic of using it in the classroom, even though some believed that it would consume time. However through this simple study, it was discovered that the teachers realised that training was essential to ensure that rekenrek would not be underused as a mere calculating tool or misused as a toy that children might play with. From the teachers' comments too, it was discovered that the training session ought to convey the underlying principles of the intended purpose of designing a mathematics manipulative so that its main aim of using it would not be overshadowed by any other manipulative that looked similar to it but did not function in the same way.

### **Implications and Recommendations**

Since counting is one of the basic and critical skills required in learning mathematics, the use of manipulative like numeral cards, colourful objects, measuring instruments, geometric models and Geoboard have long been incorporated to support understanding among young children. Rekenrek, on the other hand, overrides the advantages offered by these other manipulative as its design encourages children's to develop the mathematical skills like automaticity, subitisation, flexibility, cardinality, and decomposition, that are essential to counting. However, despite the complexity and the intricate mathematically designed features, the real 'strengths' of rekenrek to develop pupils' counting skills will remain untapped if proper and adequate training to the teachers is not given. The findings of this paper further contributes to the existing body of knowledge on the critical needs of designing and developing a hands-on training session that allows teachers to investigate and discover the real purpose behind the design of rekenrek. For, if rekenrek is used without any proper training, the essence of its mathematical design will be lost as it will be treated merely like an abacus by the educators and like a colourful toy to play with by the pupils.

An important implication of this study is that at the end of a training programme on using a manipulative, teachers have to carry back a deep understanding of the underlying principles of using it in the classroom as part of developing a lesson. This is to prevent any disillusion in effectively delivering a lesson as discovered in this study where a participant falsely believed that rekenrek cannot be used to develop multiplication. A method of checking that the teachers have clearly and correctly understood the proper use of a manipulative need to be adopted and the simplest way, of course through written or oral responses at the end of a training programme. This can be a very simple but effective way of clarifying any misconception or false allegations before they return to their classrooms. Since this study only looks at the teachers' perception before using rekenrek in the classroom, future studies can explore these teachers' perception after conducting their lessons using rekenrek. It will be interesting to investigate their enthusiasm level before and after using rekenrek in the classroom, with reality setting in after the practical use of rekenrek in guiding pupils to explore and re-invent their understanding of addition and subtraction.

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