Numeracy, Literacy and Newman’s Error Analysis

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Newman (1977, 1983) defined five specific literacy and numeracy skills as crucial to performance on mathematical word problems: reading, comprehension, transformation, process skills, and encoding. Newman’s Error Analysis (NEA) provided a framework for considering the reasons that underlay the difficulties students experienced with mathematical word problems and a process that assisted teachers to determine where misunderstandings occurred. NEA also provided directions for where teachers could target effective teaching strategies to overcome them. NEA experienced a reawakening in Australia and has been included in a number of programs such as the Counting On program in the Australian state of New South Wales. This paper presents findings of a pre-post test given to 1213 students participating in the 2008 Counting On program and examines NEA as a diagnostic tool linking numeracy and literacy and will discuss how teachers have also used NEA as a remediation and general classroom pedagogical strategy for primary and secondary schools.

Background

While most mathematical questions involve the use of words, not all are classed as word problems. A primary condition of word problems is the inclusion of a word description of a context within which the problem resides such as the word problem shown in Figure 1.

Paul went on a bike hike. He rode 402 km on his bicycle over 6 days. He rode the same distance each day. How far did Paul ride each day?

Figure 1. A typical word problem.

Key words: Newman’s Error Analysis; Problems; Errors; Numeracy; Literacy
Mathematical word problems and their place and importance in the school curriculum have attracted diverse opinions. “Teachers seem not to like word problems. Many have asked me why these are used to ‘trick’ children in assessments” (Askew, 2003, p. 78). It is well recognised that students appear to struggle with both the literacy and mathematical demands of typical mathematical word problems.

... at the upper primary level most errors on mathematics tests and examinations are caused by Reading, Comprehension or Transformation errors, or by Carelessness. Often, pupils are able to carry out one or more of the four operations (+, -, x, ÷) needed to answer a question, but they do not know which operations to use (Clements, 2004, p. ii).

The importance of word problems, it is argued, lies in the centrality of language in the teaching and learning of mathematics (Clements & Ellerton, 1993). Others would also argue that a deeper level of mathematics is needed beyond procedural proficiency, and that a conceptual knowledge of mathematics is the goal (Carpenter & Lehrer, 1999). Some would maintain that language provides a vehicle for rich classroom discussions and assists teachers and students to appreciate the power of mathematics in making sense of their world.

... if the essence of mathematics is the setting up of and working with mathematical models, and if we treat word problems in such a way, then they might have a role to play in helping children better understand the process of mathematizing. And with the increasing mathematizing of the world (from national test scores to pension prospects), informed and critical citizens need to be aware that mathematizing is not something that arises from the world, but something that is done to the world. In a small way, working on word problems might help begin to develop this awareness (Askew, 2003, p. 85).

So in summary, while the language demands of the mathematics curriculum are important and need to be developed, they also contribute to the difficulties experienced by students struggling with mathematics. Thus mathematics teachers must be aware of the literacy and numeracy issues involving word problems. But surely this cannot be of great importance because students who reach upper primary school and early secondary school can read, calculate and write. This may not be the case for all students.
In Australia there has been an increasing concern with students in the middle years who are struggling with mathematics. Gervasoni, Hadden and Turkenburg (2007) conducted a large study of number learning in 2006 of over 7000 children in Ballarat in the Australian state of Victoria for the purpose of identifying issues that could inform the development of a professional learning plan. A notable number of students (31%) beginning Grade 6 were found not yet able to read, write, order, and interpret four-digit numbers nor use reasoning-based strategies for calculations in addition, subtraction, multiplication and division.

In New South Wales the Department of Education and Training (NSWDET) in responding to similar findings implemented the Counting On Program to address the needs of students who were excluded from effective mathematics study in the middle years and beyond because of a lack of understanding of and proficiency with the early school mathematical knowledge. The Counting On program was designed with a twin learning focus upon student and teacher learning and has continued to expand and evolve. The initial program was designed for first year secondary school students (Year 7) who had not achieved specific New South Wales Stage 3 (primary school) mathematics outcomes by the time they commenced secondary school. It was later extended to include the primary schools and now targets the middle years (9-14 year olds).

The Counting On program has a strong research base starting with the Counting On Numeracy Framework (Thomas, 1999) which was an extension of work by Cobb and Wheatley (1988), Beishuizen (1993), Jones, Thornton, Putt, Hill, Mogill, Rich and van Zoest (1996) and relates to the Count Me In Too Learning Framework in Number (LFIN; Wright, 1998; Wright, Martland, & Stafford, 2000). This theoretical base was supported by an increasing number of Counting On evaluation studies (Mulligan, 1999, Perry & Howard, 2000, 2002, 2003; White 2008, 2009).

In 2007 the program underwent a major revision and was implemented in 122 schools across NSW which were grouped into 30 clusters with each cluster supported by a mathematics consultant. It was based on the earlier Counting On models but included changes designed to simplify and encourage further and ongoing involvement of schools. One of the features of the revised model was the inclusion of Newman’s Error Analysis (NEA).
Communicating is one of the five processes contributing to the Working Mathematically strand in the NSW mathematics school curriculum. Students are expected to learn to use appropriate language and representations to formulate and express mathematical ideas in written, oral and diagrammatic form. Thus the inclusion of NEA aimed to address the difficulties students were experiencing with mathematical word problems and the problems teachers were experiencing with their students’ difficulties. The success of the inclusion of NEA is the focus of this paper. The next section will present a brief history of NEA, followed by the results of using NEA in a remedial numeracy program for struggling middle years students.

Newman’s Error Analysis

In the 1980s and 1990s NEA was mainly promoted in Australia by Clements (1980, 1982, 1984) and in collaboration with Ellerton (e.g., Clements & Ellerton, 1992, 1993, 1995; Ellerton, & Clements, 1991, 1996, 1997) although there were others (e.g., Casey, 1978; Clarkson, 1980; Watson, 1980; Tuck, 1983; Faulkner, 1992). NEA also spread widely throughout the Asia-Pacific region such as in Brunei (Mohidin, 1991); in India (Kaushil, Sajjin Singh & Clements, 1985); in Malaysia (Marinas & Clements, 1990; Clements & Ellerton, 1992; Sulaiman & Remorin, 1993); in Papua New Guinea (Clements, 1982; Clarkson, 1983, 1991); in Singapore (Kaur, 1995); in the Philippines (Juminez, 1992); and in Thailand (Singhatat, 1991; Thongtawat, 1992).

This initial momentum declined in New South Wales and NEA had almost disappeared and its inclusion in the Counting On program in 2007 was serendipitous. Clements had moved to the University of Brunei Darussalam and designed and implemented a national professional learning program for primary teachers titled, Active Mathematics In Classrooms (AMIC; White & Clements, 2005). The program targeted numeracy and had nine specific aspects, of which NEA was one. Six of the aspects of the AMIC program were reported in the NSW primary school journal, Square One, for the Mathematics Association of New South Wales. An article on the use of NEA was selected and added to the teacher reader section of the NSWDET’s website in 2006 (White, 2005). This article created a renewed interest by teachers in NEA and it was subsequently added to the Counting On program in 2007.
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The reasons for the inclusion of NEA, (Newman, 1977, 1983) in the 2007 and 2008 programs were to assist teachers when confronted with students who experienced difficulties with mathematical word problems. It challenged the prevailing practice of giving students ‘more of the same’ involving drill and practice in the hope that the students would rectify their difficulties. NEA provided a framework for considering the reasons that underlay the difficulties and a process for assisting teachers to determine where misunderstandings occurred and where to target effective teaching strategies to overcome them. Moreover, it provided excellent professional learning program for teachers and made a nice link between literacy and numeracy.

NEA was designed as a simple diagnostic procedure. Newman (1977, 1983) maintained that when a person attempted to answer a standard, written, mathematics word problem then that person had to be able to pass over a number of successive hurdles: Level 1 Reading (or Decoding), 2 Comprehension, 3 Transformation, 4 Process Skills, and 5 Encoding (see Table 1 for the interview prompts). Along the way, it was always possible to make a careless error and there were some who gave incorrect answers because they were not motivated to answer to their level of ability. Newman’s research generated a large amount of evidence highlighting that far more children experienced difficulty with the semantic structures, the vocabulary, and the symbolism of mathematics than with the standard algorithms. In many Newman studies carried out in schools the proportion of errors first occurring at the Comprehension and Transformation’ stages has been large (Marinas & Clements, 1990; Ellerton & Clements, 1996; Singhatat, 1991). Thus, studies regularly reported that approximately 70 per cent of errors made by Year 7 students on typical mathematics questions were at the Comprehension or Transformation levels. These researchers also found that Reading (Decoding) errors accounted for less than 5 per cent of initial errors and the same was true for Process Skills errors, mostly associated with standard numerical operations (Ellerton & Clarkson, 1996). Also, Newman’s research consistently pointed to the inappropriateness of many remedial mathematics programs in schools in which the revision of standard algorithms was overemphasised, while hardly any attention was given to difficulties associated with Comprehension and Transformation (Ellerton & Clarkson, 1996). There have been adaptations and two procedures that modified the interview procedures used by Newman (1977) will now be briefly described.
**Table 1**

*The Newman’s Error Analysis Interview Prompts*

<table>
<thead>
<tr>
<th></th>
<th>Prompt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Please read the question to me. If you don’t know a word, leave it out.</td>
</tr>
<tr>
<td>2</td>
<td>Tell me what the question is asking you to do.</td>
</tr>
<tr>
<td>3</td>
<td>Tell me how you are going to find the answer.</td>
</tr>
<tr>
<td>4</td>
<td>Show me what to do to get the answer. “Talk aloud” as you do it, so that I can understand how you are thinking.</td>
</tr>
<tr>
<td>5</td>
<td>Now, write down your answer to the question.</td>
</tr>
</tbody>
</table>

**Analysis of All Levels**

The first adaptation by Casey (1978) in a study of the errors made by 120 Grade 7 students in a single high school instructed the interviewers to help students over errors. If a pupil made a Comprehension error, the interviewer would note this and explain the meaning of the question to the pupil, and this process would continue until the student had answered the question. Thus, in Casey’s study, a pupil could make a number of errors on the one question and thus it is difficult to compare Casey’s interpretations with Newman’s. However, Casey’s method was attractive to teachers who were more interested in how the students performed at the Process level.

**Analysis of All Answers**

The second adaption was proposed by Ellerton and Clements (1997) who used a modified form of the Newman interview method to analyse the responses by students in Grades 5 through 8 to a set of 46 questions. All responses, both correct and incorrect, were analysed. A correct answer which, after analysis, was not deemed to be associated with an adequate understanding of the main concepts, and/or skills and/or relationships tested by a question, would be associated with a Newman error category, even though the answer was correct. Ellerton and Clements’ modification led to the adoption of a slightly different definition of “Careless” error from that previously given by Clements (1982).

This concern with linking correct answers as equivalent to understanding has been researched in other contexts. For example, Ellerton and Olson (2005) conducted a study of 83 Grades 7 and 8 American students completing a test comprising items from Illinois Standards Achievement Tests. Their findings reinforced the fact that students’ scores on tests do not necessarily reflect their level of understanding of mathematical concepts and
relationships. The results indicated a 35% mismatch with students who gave correct answers with little or no understanding and others who gave incorrect answers but possessed some understanding. The authors cast doubt on the use of large scale testing programs as a means of making comparisons or being used as basis for the allocation of resources.

While there are other theoretical approaches available to teachers, NEA offers one of the easiest to use and adapt and has proven popular among NSW teachers for the ease of the diagnostic features. What is also surprising is how NEA has been used by teachers as a problem solving strategy for students and as a classroom pedagogical strategy. In the next section, data from the 2008 evaluation report (White, 2009) will examine the student learning outcomes and the teacher use of NEA.

Student Learning
The 2008 Counting On program was implemented in 99 schools across the state. An assessment instrument based on the learning framework was administered by the class teacher as a whole class schedule covering place value, addition, subtraction, multiplication, division, and word problem tasks. The assessment schedule results were used by the teacher to identify the student target group. The target group completed the program and were tested at the start and finish of the program. The teachers were asked to record the results of the target group assessment process involving a minimum of 5 students per class on an excel spreadsheet supplied to them. The spreadsheet recorded the initial level on the learning framework and NEA for the students before the program was implemented and again following 10 weeks of targeted activities.

The Counting On program is funded under an Australian federal government program and there is a mandatory evaluation process that includes instruments and reporting requirements. The Counting On program also has to report to other NSW state bodies and other data is collected for these purposes. The author of this paper was given all the data collected from all the instruments used and asked to analyse and construct an evaluation report. He had neither input into the design of these instruments nor the collection of data although he was able to collect further data. Thus there are methodological issues that arise such as a concern with the initial and final student level diagnosis by teachers. While the facilitators are trained in the use of NEA, there are concerns with the process involving the other
teachers and this will be discussed later in this paper. However as a result of these concerns with the integrity of some of the data, it was decided to use only simple statistical tools in the analysis.

In 2008 data was collected from 74 schools with 55 primary schools, 16 secondary schools and three central schools. There were 1213 students with 954 primary students (78.6%) and 259 secondary students (21.4%). Only one of the two questions involving Newman’s Error Analysis in the assessment instrument was recorded for each student. The NEA scale from 1 to 5 was used, and a category 6 was added to represent those who could complete the word problem successfully. Table 2 displays the initial and final NEA levels for the 2008 cohort and indicates an improvement in the overall levels from the initial to the final student assessments.

Table 2
The Initial and Final Newman’s Error Analysis Levels

<table>
<thead>
<tr>
<th>NEA Levels</th>
<th>Initial Level Frequency</th>
<th>Initial Level Frequency %</th>
<th>Final Level Frequency</th>
<th>Final Level Frequency %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>196</td>
<td>16.2%</td>
<td>51</td>
<td>4.2%</td>
</tr>
<tr>
<td>2</td>
<td>452</td>
<td>37.3%</td>
<td>234</td>
<td>19.3%</td>
</tr>
<tr>
<td>3</td>
<td>399</td>
<td>32.9%</td>
<td>477</td>
<td>39.3%</td>
</tr>
<tr>
<td>4</td>
<td>101</td>
<td>8.3%</td>
<td>220</td>
<td>18.1%</td>
</tr>
<tr>
<td>5</td>
<td>37</td>
<td>3.1%</td>
<td>134</td>
<td>11.0%</td>
</tr>
<tr>
<td>6</td>
<td>28</td>
<td>2.3%</td>
<td>97</td>
<td>8.0%</td>
</tr>
<tr>
<td>Total</td>
<td>1213</td>
<td>100.0%</td>
<td>1213</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Table 3 shows that the majority of students have improved by 1 or more levels (56.6%), with a sizeable group improving two levels (15.6%). There are a small group of students who improved by 3 and 4 levels as there are some who decline by 1, 2 or more levels.

The descriptive statistics record an increase in the mean from 2.52 for the initial level (SD = 1.096) to 3.37 for the final level (SD = 1.254). Using a paired sample T-Test, the results indicate that the change in the student outcomes for mathematical word problem levels at the start and finish of the 10 week Counting On 2008 program was statistically significant.
While the 2008 data collected for the pre and post program student learning outcomes indicated that a statistical significant increase existed in the NEA levels, of concern was the group who did not show an increase. In a short program as this it is unrealistic to expect that all students will make great leaps on the NEA levels. These targeted students have been struggling for some time with their mathematical and literacy difficulties and have developed judgements of their own ability. To improve 1 level, especially for the NEA scale which could involve the improvement of reading or comprehension, is quite remarkable in such a small time frame.

However, there may be other possible explanations for the lack of improvement in a small group of students or the apparent decline in others. Vaiyatvutjamai and Clements (2004) analysed the errors made by 231 Form 3 (Year 9) Thai students in two Chiang Mai government secondary schools. Students completed tasks before and immediately after a series of 13 lessons. A number of misconceptions were revealed and although some were clarified as a result of the lessons, there were others that remained and seemed to be ‘fossilised’. A ‘fossilised misconception’ was used to denote the situation where a student maintains a faulty conception despite having been specifically taught the ‘official’ defining characteristics of the relevant concept. Associated with this then is the absence of cognitive change over time or even resistance to change over time, so that cognitive inertia persists despite the individual having been taught the ‘proper’ view of the concept.
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The implications for this current study are that the strategies and procedures of the intervention program of Counting On should become integrated into the everyday classroom and continue after the program has finished. These ‘fossilised misconceptions’ may require a greater time period for them to be changed.

Also of interest is that while the study by Vaiyatvutjamai and Clements (2004) involved students across the range of abilities, the results for low performing students challenged the use of the term misconception for many of the student errors. “A misconception can be regarded as a fairly stable, but inappropriate, way of thinking ...analysing the errors made by low performers in this study, was that the word ‘stable’ was not one that could sensibly be used” (p. 181). Students with ‘unstable’ conceptions will give different answers at different times and hence it is possible that their test scores will decline. Students who have not developed confidence in their ability to answer a question may revert to guessing. This may well explain the 35% mismatch with students who gave correct answers with little or no understanding and others who gave incorrect answers but possessed some understanding reported in Ellerton and Olson’s (2005) study. While students using a guessing strategy may cause some instability in the results for the process level in NEA if not identified by the teacher, it would be easily revealed by the second adaption of NEA (Ellerton & Clements, 1997) where correct answers are interrogated.

This completes the section on NEA and student learning and the next section will discuss the second learning focus which is upon the teacher. Teachers were shown how to use the NEA prompts (see Table 1) to diagnose the difficulties that their students were having with mathematical word problems. The following section of this paper will explore how teachers responded to NEA as an aspect of the Counting On program.

Teacher Classroom Use

The professional learning of teachers within the Counting On program had evolved by 2007 into a two-day conference attended by one or two volunteer teachers from each school who would act as facilitators. The facilitators would then return to their school, form and train a team, and then implement the program. While initially a ‘train the trainer’ model may have been a loosely accurate description of this process, it evolved and now a better term would be to call it a ‘facilitated model’ of teacher professional learning.
Whereas in a ‘train the trainer’ process there would be an expectation of a consistency in delivery with everyone trained and then delivering the training in the same way, a facilitated model allows for more variability with regards to how the program is implemented because the expectation is that the facilitator will shape the program to meet local needs. This change meant that whereas cascade models of train the trainer suffered from ‘dilution’ as the process moves from level to level, in contrast a facilitated model had the potential to be both better and worse than the original facilitator training. The evaluation reports highlighted that the success of the program depended to a great extent upon the school facilitator. Thus the implementation of NEA depended upon the school facilitator.

The 2007 and 2008 evaluation reports (White 2008, 2009) revealed that the majority of teachers were strongly positive about the inclusion of NEA into the program. Teachers reported it being an understandable, easy to use, framework and process for uniting numeracy and literacy and there were requests for further opportunities for teacher professional learning involving NEA.

I probably found this to be where I gained the most knowledge. The children having the most difficulty with maths problems were usually the ones with poor literacy skills and using the analysis pinpointed exactly where the problem was. I have used the prompts regularly as a teaching strategy and introduced it to other staff members who are also incorporating it in their classrooms (White, 2009, p. 51).

In the 2007 evaluation document it was reported that there was a divide between the responses involving primary and secondary teachers. NEA appeared to resonate more easily with primary teachers and with the issues of ‘numeracy across the curriculum’ and ‘every teacher being a teacher of literacy’ that are promoted by the NSWDET. Primary teachers were able to use it to analyse their Basic Skills Test errors (schools receive a report on their students who sat for the NSW state wide primary school testing program) and develop strategies to improve their students’ literacy needs. In the secondary school, the resonance was not as high, resulting in some secondary teachers regarding NEA as an issue that was not their concern.
The 2007 report stated:

A typical comment extract was ‘The inclusion of NEA has been extremely beneficial in providing teachers with new insights into where and why the students break down in solving word number problems. The workshops we have provided have indicated that a number of secondary mathematics teachers find it difficult to embrace this process’. However it should be emphasised that this does not represent all secondary teachers, as is evidenced by the following comment ‘One head teacher has adopted/adapted it to assist senior students in Stage 6 mathematics’ (White, 2008, p. 12).

However there were comments in the 2008 report involving secondary teachers that pointed to a deeper entrenched attitude that school facilitators had to address when implementing the NEA procedure:

Teachers found this interesting and even though they were confronted with evidence of what they already claim i.e. it is not the maths that is causing the child difficulty in the question but the literacy, many teachers are not prepared to tackle teaching reading and understanding the question, or finding pertinent information within the question (White, 2008, p.39).

The majority accepted the challenge of incorporating literacy issues within a mathematics lesson and the 2008 evaluation report described how teachers had extended the use of NEA beyond a diagnostic tool to a pedagogical and remedial tool.

The five Newman’s prompts were displayed on a poster in the classroom (see Figure 2) and were referred to in whole class, small group and individual student interactions. All students were expected to work through the NEA levels for all mathematical questions. In a whole class setting, sometimes the teacher selected students who worked aloud in order to scaffold the learning of those struggling with one of the levels. Sometimes the whole class would work aloud together through the prompts for a question, with the teacher interrupting to probe each response in order to assist students to construct a deeper understanding of the question, and the process for finding a correct answer. The students would then construct an answer to the question in their work books.
The manner in which teachers used the NEA prompts as a generic problem solving approach is reflected in the following teacher comments:

*The Newman’s error analysis and follow-up strategies have helped students with their problem-solving skills, and teachers have developed a much more consistent approach to the teaching of problem-solving. Not only has it raised awareness of the language demands of problem solving, but through this systematic approach, teachers can focus on teaching for deeper understanding (White, 2009, p. 37).*

Many primary teachers told of how it had been adapted across their different subjects and their different student year stages.

*Groups are differentiated to cater for learning abilities. My Y5/6 children all participate regularly in ability based maths groups within my room. They analyse their own learning often through learning logs. Children practice NEA with whole group problem solving at beginning of lessons (not always, but regularly). Children are doing more maths, but maintaining engagement for entire hour and 25 mins. Maths lessons are much more dynamic! (White, 2009, p. 47).*
To conclude this section, the evaluation reports indicated that the inclusion of NEA in 2007 was welcomed by teachers and this positive reaction was also reported by the 2008 teachers involved with the Counting On program, as the following teacher comment indicates:

*This is the best aspect of the programme. I now use the steps as a teaching strategy for those with difficulties in my classes. Going through the questions each time helps the students with difficulties at different levels. I have the questions on a poster in my class. I have also started talking to the English department about getting some help with certain students (White, 2009, p. 50).*

**Conclusion**

The Counting On program was a success in improving both teacher and student learning outcomes through the inclusion of NEA. The data revealed a statistical and educationally significant improvement existing in student learning outcomes between the start and the completion of the Counting On program involving mathematical problem solving using word problems. As well NEA was being used by teachers as a remedial classroom strategy and as a wider classroom pedagogical strategy. Thus this article concludes that the inclusion of NEA was a powerful classroom diagnostic assessment and teaching tool for assessing, analysing and catering for student experiencing difficulties with mathematical word problems.

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